Field Guide to

Linear Systems in Optics

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SPIE Field Guides
Volume FG35

John E. Greivenkamp, Series Editor

SPIE PRESS
Bellingham, Washington USA

Library of Congress Preassigned Control Number Data

Tyo, J. Scott, author.
Field Guide to Linear Systems in Optics / J. Scott Tyo and Andrey S. Alenin
pages cm. – (SPIE Field Guide series; FG35)
Includes bibliographical references and index.
ISBN 978-1-62841-547-6
1. Linear systems. 2. Optics. 3. Fourier transformations. I. Title.
QC355.2 2015
535–dc23
2014958708

Published by
SPIE
P.O. Box 10
Bellingham, Washington 98227-0010 USA
Phone: 360.676.3290
Fax: 360.647.1445
Email: Books@spie.org
www.spie.org

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Printed in the United States of America.
First printing.

Introduction to the Series

Welcome to the *SPIE Field Guides* — a series of publications written directly for the practicing engineer or scientist. Many textbooks and professional reference books cover optical principles and techniques in depth. The aim of the *SPIE Field Guides* is to distill this information, providing readers with a handy desk or briefcase reference that provides basic, essential information about optical principles, techniques, or phenomena, including definitions and descriptions, key equations, illustrations, application examples, design considerations, and additional resources. A significant effort will be made to provide a consistent notation and style between volumes in the series.

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The *SPIE Field Guides* are intended to be living documents. The modular page-based presentation format allows them to be easily updated and expanded. We are interested in your suggestions for new *Field Guide* topics as well as what material should be added to an individual volume to make these *Field Guides* more useful to you. Please contact us at **fieldguides@SPIE.org**.

John E. Greivenkamp, *Series Editor*
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The College of Optical Sciences (OSC) at the University of Arizona has long offered a course called "OPTI512R: Fourier Transforms, Linear Systems, and Optics" in its graduate program. The course was initiated and designed by Prof. Jack Gaskill, and was taught largely out of a textbook by the same name that was published in 1978. When Prof. Tyo joined OSC in 2006, he was asked to take over the course, as Prof. Gaskill had retired some years earlier.

Tyo came to the class with an electrical engineer's classic understanding of linear systems in time and frequency. Tyo quickly came to realize that, at that time, Prof. Gaskill's textbook was the only one written from the perspective of an optical engineer who needs to take 2D spatial Fourier transforms instead of 1D temporal ones. This difference gives rise to several subtle but important stylistic requirements that Prof. Gaskill captured well in his text. As with most instructors, Tyo began to add his own take on the material over the years.

Andrey Alenin joined his group in 2010, and he showed a strong interest in both the pedagogy and the presentation of the course material; the two authors have since worked together to refine the presentation. As of the writing of this *Field Guide*, Prof. Gaskill's text is still the primary reference in the class. However, when John Greivenkamp discussed with us the possibility of writing a *Field Guide* on this topic, he gave the authors the opportunity to go through the notes and reorganize them into a sequence more suited for this handbook format.

The process is, of course, circular. During the current semester of teaching OPTI512R, while completing this *Field Guide*, the authors have realized that the entire structure of the course will need to be revised going forward. The efforts undertaken to write this book have provided a new perspective on the classic course content.

We would like to extend our gratitude to the following individuals who aided in the preparation of parts of this book. Series editor John Greivenkamp was invaluable for his guidance on style and his tips about what to include and what to omit. Brian Anderson from the University of Arizona read and commented on several of the pages that discuss topics from quantum mechanics. Scott McNeill from SPIE was of help in setting up the formatting of the book.

We are grateful to the owners and staff of the Cartel Coffee Lab and the Dragoon Brewery, who allowed us to occupy power outlets, seats, and their Wi-Fi connections for hours on end as we tried to escape the campus and hide in order finish the book.

Prof. Tyo would like to express his gratitude to his wife, Elizabeth Ritchie, for her patience while he worked on the book during their sabbatical.

Andrey Alenin would like to thank Geraldine Longo for her continuous encouragement, as well as comments and advice on aesthetics of presentation.

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Table of Contents

Table of Contents

Table of Contents

Glossary

Functions in this *Field Guide* are functions of spatial variables x and y unless noted otherwise. Lowercase letters are used to denote functions of the spatial variables ($f(x), g(x)$), whereas capital letters represent their Fourier transforms ($F(\xi), G(\xi)$).

Sequences of discrete samples of a function are denoted with the subscript k (f_k, g_k), and samples of the corresponding DFTs are denoted with subscript n (F_n, G_n).

Primed variables (x', y', ξ', η', etc.) denote variables of integration.

BPF	Bandpass filter
CTF	Coherent transfer function
D	Pupil diameter
d_o, d_i	Object and image distances
$\mathcal{D}\{f_k\}$	Discrete Fourier transform of sequence f_k
$\mathbf{E}$	Complex vector electric field
$E[f(x)]$	Expected value of $f(x)$
f	Focal length
$f/\#$	F-number
$f/\#_w$	Working F-number
$f_s(x)$	Ideally sampled function $f(x)$
$\mathcal{F}\{f(x)\}$	Fourier transform of $f(x)$
$h(x)$	Impulse response
$H(\xi)$	Transfer function
$\mathcal{H}(\xi)$	Optical transfer function
HPF	High-pass filter
$J_0(x)$	Zeroth-order Bessel function of first kind
$\mathbf{k}$	Wave vector
L	Spatial extent of a function
$\mathscr{L}$	Linear shift invariant operator
$\mathcal{L}\{f(t)\}$	Laplace transform of $f(t)$
LPF	Low-pass filter
$m_n(f(x))$	n^{th} moment of $f(x)$
MTF	Modulation transfer function
OTF	Optical transfer function
PSD	Power spectra density
PSF	Point spread function

Glossary

r	Polar coordinate radius
$\mathbf{r}$	2D vector $x\hat{\mathbf{x}} + y\hat{\mathbf{y}}$
$\mathscr{S}$	Mathematical operator
SNR	Signal to noise ratio
T	Temporal period
$t(x,y)$	Transmission function
u	Complex scalar optical field amplitude
W	Spatial frequency bandwidth
$W(x,y)$	Wavefront aberration function
X	Spatial period
x, y	Cartesian coordinates
x_s	Spatial sampling interval
$\mathcal{Z}\{f_k\}$	Z-transform of sequence f_k
$\delta(x - x_0)$	Impulse function at $x = x_0$
Δx	Sampling resolution in the space domain
$\Delta \xi$	Sampling resolution in the frequency domain
$\gamma_{fg}(x)$	Correlation between functions $f(x)$ and $g(x)$
$\gamma_x, \gamma_y, \gamma_z$	Direction cosines
η	Spatial frequency in y
λ	Wavelength
ν	Temporal frequency
θ	Polar coordinate angle
ρ	Radial distance in frequency plane
$\boldsymbol{\rho}$	$\xi\hat{\boldsymbol{\xi}} + \eta\hat{\boldsymbol{\eta}}$
υ	Normalized frequency ξ/ξ_s
ξ	Spatial frequency in x