

**Modeling the
Optical and Visual
Performance of the
Human Eye**

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Pier Giorgio Gobbi

**SPIE
PRESS**

Bellingham, Washington USA

Library of Congress Cataloging-in-Publication Data

Gobbi, Pier Giorgio.

Modeling the optical and visual performance of the human eye /

Pier Giorgio Gobbi.

p. cm.

Includes bibliographical references and index.

ISBN 978-0-8194-9254-8

1. Physiological optics. 2. Eye—Mathematical models. I. Title.

QP475.G48 2013

611'.84—dc23

2012022682

Published by

SPIE

P.O. Box 10

Bellingham, Washington 98227-0010 USA

Phone: +1 360.676.3290

Fax: +1 360.647.1445

Email: Books@spie.org

Web: <http://spie.org>

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Printed in the United States of America.

First printing



In grateful memory of

CARLO ALBERTO SACCHI (1937–1989)

PASCAL ROL (1956–2000)

GIAN PIERO BANFI (1946–2002)

three smart scientists, three good friends

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Preface

- Can a schematic eye model reproduce the foveal images recorded in human eyes, and if so, to what degree of accuracy?
- To accomplish this, is it necessary to develop a new eye model?
- What is the physical approach required?
- What is the optimum defocus that corresponds to the maximum performance of ocular optics?
- What is the typical performance of ocular optics at different pupil sizes, and how far is it from the diffraction limit?
- How do spherical and chromatic aberration affect optical performance of the human eye?
- What is the ultimate optical performance of the eye?
- Do the international standards on safety from optical radiation properly estimate retinal irradiance?
- Can the optical performance of an emmetropic eye be improved further by means of optical aids or refractive surgery?
- Can a neurophysical model of the human eye simulate its visual performance with satisfactory accuracy?
- How can human visual performance be characterized quantitatively beyond visual acuity?
- What is the typical visual performance of an average human eye on axis in different visual conditions?
- Which are the most relevant optical factors limiting human visual performance?
- What is the ultimate visual performance of the eye?
- Is it possible to enhance the visual performance of the human eye through either optical aids or surgery?
- Is the existence of spatial frequency channels compatible with the neurophysical model here developed?

These basic questions are addressed in this book from a deterministic approach. Quantitative answers are given through the development of physical models that describe the optical process of image formation on the fovea, and the subsequent neural processing of visual information gathered by photoreceptors.

A faithful and robust simulation of the optical and visual performance of the human eye is provided for axial vision of distant objects in a variety of visual conditions. The book moves from intrinsically theoretical aspects (optical and neurophysical models of the eye) to include a large number of experimental measurements from within scientific literature. The model parameters are tuned to the observed phenomenology, in order to validate the predictive power of the models. The results turn out to be very satisfactory in terms of quantitative and qualitative adherence of the model predictions to field measurements.

The majority of material in this book is original and is the result of investigations made by the author during the last decade. The most relevant achievement of this work is the capacity to evaluate visual acuity for a range of visual conditions, such as variations in pupil size, refractive error, and ambient illumination.

The material is organized into two parts: optical and neurophysical aspects of the eye model. Each part is then divided into two sections. The first sections are devoted to assessment of the specific models through derivation of parameters from the best-fitting of experimental data. The second sections contain descriptions of the relevant properties derived from the models, together with discussions and connections to real-life situations. The reader should note that chapters and paragraphs with high-level mathematical and physical optics content can be safely skipped without compromising overall comprehension. To this end, a brief summary is provided at the end of each chapter.

Part IA defines the optical eye model that is used throughout the book—the chromatic aspherical Gullstrand exact (CAGE) eye model, which is developed from the Gullstrand exact eye model with the introduction of aspherical interfaces and chromatic index dispersion. Surface asphericities are derived from the best-fitting of line images recorded in a classical double-pass experiment, with similar images obtained from the CAGE model. Theoretical modeling of the double-pass experiment requires a complex physical optics analysis, including directionality of foveal reflection and spatial partial coherence of illumination light. The procedure is supported by the available accurate reporting of experimental conditions. The result is an excellent match-up of model predictions with measurements at all pupil sizes ($R^2 > 0.92$). The values of surface asphericities match well with independent measurements performed *in vivo*.

Part IA demonstrates the feasibility of using schematic eye models not only for estimating first-order geometrical optics properties and aberrations, but also for evaluating and reproducing the actual retinal images recorded by human eyes with high accuracy. The physical optics approach is attractive, since the starting point for the calculation is not the

usual wave aberration at the exit pupil (estimated from aberration data), but a well-defined optical scheme. This approach allows for the joint treatment of monochromatic and chromatic aberrations, as well as diffraction. As a consequence, the CAGE model is representative of the average human eye for distance foveal imaging.

Part IB provides a detailed presentation of optical performances exhibited by the CAGE model. The model's paraxial properties at the central wavelength coincide with those of the Gullstrand exact model, but vary with wavelength. The CAGE eye model is characterized through the analysis of spherical aberration, point and line spread functions at variable pupil sizes, relative energy content, and modulation transfer function. Single-valued parameters are extracted for a simpler, direct description of optical behavior, including Strehl and Struve ratios, optimum defocus, full widths at half maximum for point and line images, spatial frequency bandwidths, and retinal gain. The entire characterization is illustrated by the continuous comparison between monochromatic and white light performances, as well as by comparison with two diverging behaviors: the diffraction-limited model and the purely spherical model (Gullstrand exact). CAGE model predictions are successfully compared with independent *in-vivo* measurements of spherical aberration and psychophysical modulation transfer function.

The most important innovative contributions from Part IB are as follows:

- Optimum defocus is effective in maximizing the foveal performance against spherical aberration (explaining the hyperopic choice operated by Gullstrand in his model).
- Retinal gain in conditions of optimum defocus is much larger than that assumed in international standards for laser safety.
- Chromatic aberration is the major limiting factor of optical performance.
- The eye behaves as a poor optical system in monochromatic illumination, but in white light it performs only 50% worse than a diffraction-limited eye.

In Part IIA, the CAGE optical eye model is merged with a neurophysical model of the eye from Barten, which describes the psychophysical response of the eye to sinusoidal bar stimuli with variable frequency, contrast, and luminance (ocular contrast sensitivity). The Barten model is based on the estimate of noise level generated internally in the eye. It depends on a few scalar parameters related to the integration properties of the eye, and on the ocular modulation transfer function. Modifications to the original Barten model have been introduced for physical consistency and improved phenomenological representation. The main modification

involves the modulation transfer function of the eye, which is calculated by means of the CAGE optical model. The joint CAGE-Barten model can provide estimates of the contrast sensitivity function (CSF) for a wide range of ambient and subject conditions. Values of the model parameters are derived from the best-fitting of 15 experimental data series on CSF, taken from the literature. The overall agreement obtained is excellent ($R^2 > 0.96$), providing good predictability in a variety of test conditions.

The main achievement of Part IIA is the development of a physical model that can predict human contrast sensitivity for a large number of conditions (including pupil size and refractive error of the subject; spatial frequency, spectrum, size, and duration of the stimulus; and ambient luminance). Results are obtained by following a deterministic physical pathway, without any *ad-hoc* heuristic assumptions (as in the original Barten model). Furthermore, values of the psychophysical parameters (obtained from the best-fitting procedure) help to define both structural properties of the eye (photoreceptor quantum efficiency, neural noise spectral density) and features of the integration capability of the visual system (temporal, spatial, and frequency integration limits, lateral inhibition cutoff). Thus, the CAGE-Barten model represents an effective tool for evaluating optical and perceptive properties of the human visual system.

In Part IIB, visual performances of the CAGE-Barten model are analyzed, starting from the evaluation of the entire perceptive region in the contrast-spatial frequency plane, which characterizes the quality of vision for any visual condition. The analysis is based on two single-valued parameters—grating visual acuity and bilogarithmic area of the perceptive region—which are evaluated as a function of pupil size and pupil response, illumination spectrum, spherical aberration, defocus, stimulus properties, and psychophysical parameters. The results are satisfactorily compared with the experimental measures of Snellen visual acuity and image quality. As an example, model grating visual acuity at 3.3-mm pupil size and 160-cd/m² luminance is $-0.14 \log\text{MAR}$ (20/14.5 Snellen fraction), which well overlaps with analogous measurements performed in young subjects. The CAGE-Barten model allows analysis of visual performance in relation to the fundamental limits placed by diffraction and noise, thus quantifying potential margins of improvement. Despite being based on a single filter-detector unit, the CAGE-Barten model is compatible with the existence of a plurality of spatial frequency channels; also, fitting such channels into the CSF evaluated by the model helps to shed light on their nature and structure.

The main contribution of Part IIB is unification of the optical and psychophysical descriptions of vision under a single model, with high predictability of mean performances in the human eye. In addition

to providing access to the neural image, the model provides local and integrated metrics for the quantitative evaluation of vision quality, related to variations of observing conditions. The CAGE–Barten model represents an effective tool for reproducing and analyzing both imaging and perception behaviors of an average human eye.

I am indebted to Dr. Laura Galli, Scientific Institute Hospital San Raffaele, for precious statistical advice. I thank Prof. Gianni Gilardi, Department of Mathematics F. Casorati, University of Pavia, for providing me with useful analytical formulas. Finally, I am grateful to my wife Mara and my children Alessandra and Francesco for their confident and patient waiting for this laborious delivery.

Pier Giorgio Gobbi
Milan, Italy
November 2012