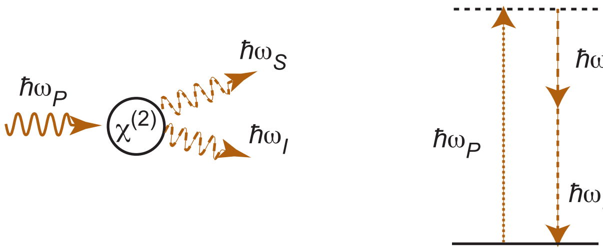
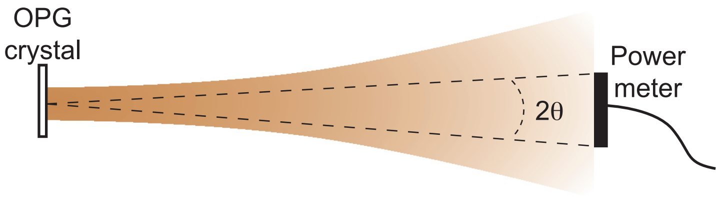
Optical Parametric Generation



Spontaneous parametric down-conversion occurs when a pump photon at ω_P spontaneously splits into two photons—called the **signal** at ω_S, and the **idler** at ω_I (by convention, the signal frequency is higher than the idler frequency, but the convention may be reversed in some contexts). The process is also called **spontaneous parametric scattering (SPS)** and **optical parametric generation (OPG)**. OPG occurs in $\chi^{(2)}$ crystals and is defined by the energy conservation statement $\hbar\omega_P = \hbar\omega_S + \hbar\omega_I$. The signal and idler central frequencies and the bandwidth are dictated by phase matching.



In the small signal regime, the power collected by a detector of a finite size is given by

$$dP_s(z) = \frac{\hbar n_S \omega_S^4 \omega_I d_{eff}^2}{2\pi^2 \varepsilon_0 c^5 n_p n_i} \frac{\sinh^2(gz)}{g^2} P_p \theta d\theta d\omega_s$$

where $$g = \sqrt{\Gamma^2 - \left(\frac{\Delta k}{2}\right)^2} \quad \text{and} \quad \Gamma^2 = \frac{2\omega_S \omega_I d_{eff}^2}{n_P n_S n_I \varepsilon_0 c^3} I_P$$

I_P and P_P are the intensity and power of the incident pump beam, respectively. Although typically weak, the OPG process can deplete the incident pump beam for high-peak-power pump beams.

Optical Parametric Oscillator

In an **optical parametric oscillator (OPO)**, a pump beam spontaneously down-converts into a signal-and-idler beam where the signal and/or idler are resonated. If the roundtrip losses are less than the parametric gain, an oscillation occurs, yielding relatively high conversion from the incident pump beam to the signal and idler. The threshold power P_{TH} for a plane-wave OPO is given by

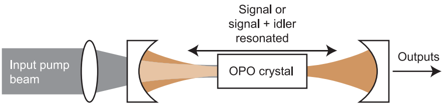
$$P_{TH} = A\frac{n_p n_S n_I \varepsilon_0 c \lambda_S \lambda_I}{4\pi^2 d_{eff}^2 L^2}\frac{(1-\rho_S)(1-\rho_I)}{\rho_S+\rho_I}$$

where A is the cross-section of the plane-wave beam, and L is the crystal length. The cavity losses are lumped together in terms ρ_S and ρ_I, given by

$$\rho_S = \sqrt{R_{aS}R_{bS}}e^{-2\alpha_S L} \quad \text{and} \quad \rho_I = \sqrt{R_{aI}R_{bI}}e^{-2\alpha_I L}$$

R_{aS} and R_{bS} correspond to the reflectivities of the two cavity mirrors, and α_S is the crystal absorption for the signal wavelength; a similar notation is used for idler variables. For an SR-OPO, $R_I = 0$.

An OPO where only the signal is resonated is a **singly resonant OPO (SR-OPO)**. An OPO where both the signal and idler are resonated is a **doubly resonant OPO (DR-OPO)**.



The OPO threshold with loosely focused Gaussian beams is given by

$$P_{TH} = \frac{n_P n_S n_I \varepsilon_0 c \lambda_S \lambda_I W^2}{32\pi L^2 d_{eff}^2}\frac{(1-\rho_S)(1-\rho_I)}{\rho_S+\rho_I}$$

$$\frac{1}{W^2} = \left(\frac{w_P w_S w_I}{w_P^2 w_S^2 + w_P^2 w_I^2 + w_S^2 w_I^2}\right)^2$$

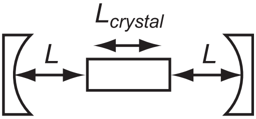
where w_P, w_S, and w_I are the beam radii for the pump, signal, and idler, respectively. For a DR-OPO, w_S and w_I are determined by the cavity. For an SR-OPO

$$1/w_I^2 = 1/w_P^2 + 1/w_S^2$$

Singly Resonant Optical Parametric Oscillator

For continuous-wave OPOs, a stable resonator and tight focusing are used to minimize the OPO threshold. The optimum focusing condition depends on the particular phase-matching interaction. Calculations of the threshold, including the effects of diffraction and walk-off, show that a focusing parameter near $\xi = 1$ (where $\xi = L_{crystal}/2z_R$) is optimum. In this region, the Gaussian-beam threshold equation gives a reasonable estimate for the **SR-OPO** threshold.

Optimum focusing for the pump laser is achieved using external optics. However, the mode size for the signal is determined by the resonator. For a symmetric linear cavity, the cavity beam waist is



$$w_o^2 = \frac{\lambda}{\pi}\sqrt{L_{eff}(R - L_{eff})}; \quad L_{eff} = L + \frac{L_{crystal}}{2n}$$

where R is the radius of curvature for the mirrors, n is the crystal index, and L is the crystal-to-mirror distance.

SR-OPO threshold calculation	Parameters
Assuming that the pump is confocally focused, $2z_R = L_{crystal}$, we calculate $w_{oP} = 62$ μm. Find w_{oS} using the cavity equation given above, yielding $w_{oS} = 80$ μm, which gives a focusing parameter $\xi_S = 0.88$. The idler beam size is determined by the pump and signal: $1/w_I^2 = 1/w_P^2 + 1/w_S^2$, which gives $w_I = 49$ μm. These beam sizes let us calculate $W = 202$ μm. From the mirror reflectivities, and assuming no crystal losses, $\rho_S = 0.95$ and $\rho_I = 0$. Placing all of the parameters into the threshold equation gives $P_{TH} = 1.1$ W.	$\lambda_P = 1.064$ μm $\lambda_S = 1.55$ μm $\alpha = 0$ (no loss) $d_{eff} = 17$ pm/V $L_{crystal} = 5$ cm $L = 3.5$ cm $n = 2.2$ Left mirror: $R_S = 100\%$ $R_I = 0$ Right mirror: $R_S = 97.5\%$ $R_I = 0$ Mirror radius of curvature = 5 cm

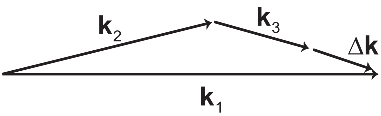
Birefringent Phase Matching

Phase matching occurs when the nonlinear polarization is in phase with the field it is driving. Phase matching for a three-wave interaction defined by the energy conservation statement $\hbar\omega_1 = \hbar\omega_2 + \hbar\omega_3$ is expressed by

$$\Delta\mathbf{k} = \mathbf{k}_1 - \mathbf{k}_2 - \mathbf{k}_3 = 0$$

where $\mathbf{k}_1$, $\mathbf{k}_2$, and $\mathbf{k}_3$ are the ***k* vectors** corresponding to the frequencies ω_1, ω_2, and ω_3, respectively. The magnitude of a k vector is $n\omega/c = 2\pi n/\lambda$, and its direction is parallel to the propagation direction. For linearly polarized fields, the index is either o or e polarized.

Phase matching occurs for $\Delta\mathbf{k} = 0$.

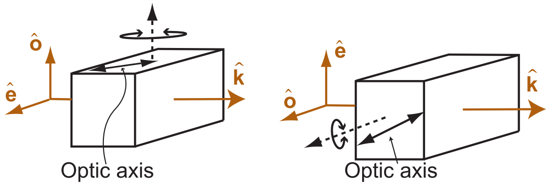


Collinear phase matching occurs when all k vectors are parallel, and **noncollinear phase matching** occurs when the k vectors are nonparallel. A noncollinear situation is shown above. If all three fields have the same polarization, and for materials with normal dispersion, perfect phase matching is not possible. Note that an absorption band lying between two of the frequencies may invalidate the normal dispersion condition. **Birefringent phase matching** uses a mixture of e and o polarizations to make $\Delta\mathbf{k} = 0$, making phase matching possible but not guaranteed. Different birefringent phase-matching types are characterized by their particular polarization combinations, most commonly categorized into **Type-I** and **Type-II phase matching**.

	ω_1	ω_2	ω_3	$\omega_1 > \omega_2 \geq \omega_3$
Type I	o	e	e	Positive uniaxial
	e	o	o	Negative uniaxial
Type II	o	e	o	Positive uniaxial
	o	o	e	
	e	e	o	Negative uniaxial
	e	o	e	

e- and o-Wave Phase Matching

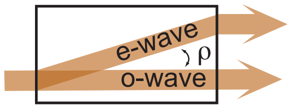
A large class of nonlinear interactions require orienting the crystal for phase matching. In *uniaxial* crystals, the phase-matching angle depends only on the angle between the Z axis and the k vector of the laser beam. This **polar angle** is usually called the **phase-matching angle**. The **crystal cut** is chosen to allow rotation of the phase-matching angle, while keeping the **azimuthal angle** (angle between X axis and k vector) fixed. For interactions confined to the principal planes of a biaxial crystal, the phase-matching angle is defined to be between one of the principal axes and the laser k vector. The mapping of an external polarization state to an e- or o-wave depends on the specific crystal orientation, as shown below:



Another important consideration when working with a mixture of e- and o-waves is **Poynting vector walk-off** ρ. In many cases, walk-off limits the effective crystal length since the interacting beams physically separate.

$$\rho = \tan^{-1}\left(\frac{n_o^2}{n_z^2}\tan\theta\right) - \theta$$

θ is the phase-matching angle.



Finding **uniaxial e- and o-waves**:

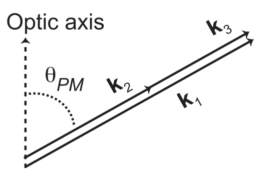
- Draw Z axis (optic axis, OA),
- Draw laser k vector,
- e-waves are polarized in the plane of OA/k-vector,
- o-waves are polarized perpendicular to this plane.

Finding **biaxial e- and o-waves**:

In the principal planes of a biaxial crystal, e-waves are polarized in the principal plane, and o-waves are polarized perpendicular to this plane.

DFG and SFG Phase Matching for Uniaxial Crystals

The table uses the notation: $n_{o1} = n_o(\lambda_1)$, $n_{o2} = n_o(\lambda_2)$, etc.; $n_{z1} = n_z(\lambda_1)$, $n_{z2} = n_z(\lambda_2)$, etc. θ is the phase-matching angle (between k and OA in figure). Only two input wavelengths are required—the third is obtained from $1/\lambda_1 = 1/\lambda_2 + 1/\lambda_3$, where $\lambda_1 < \lambda_2 \leq \lambda_3$. The notation for e and o polarizations in this guide assumes a wavelength ordering from low to high, going left to right:



***e* ↔ *o* + *o* (negative uniaxial)**	
$$\tan^2\theta = \frac{\frac{\lambda_1^2}{n_{o1}^2}\left(\frac{n_{o2}}{\lambda_2}+\frac{n_{o3}}{\lambda_3}\right)^2 - 1}{1-\frac{\lambda_1^2}{n_{z1}^2}\left(\frac{n_{o2}}{\lambda_2}+\frac{n_{o3}}{\lambda_3}\right)^2}$$	
***o* ↔ *o* + *e* (positive uniaxial)**	***o* ↔ *e* + *o* (positive uniaxial)**
$$\tan^2\theta = \frac{\frac{\lambda_3^2}{n_{o3}^2}\left(\frac{n_{o1}}{\lambda_1}-\frac{n_{o2}}{\lambda_2}\right)^2 - 1}{1-\frac{\lambda_3^2}{n_{z3}^2}\left(\frac{n_{o1}}{\lambda_1}-\frac{n_{o2}}{\lambda_2}\right)^2}$$	$$\tan^2\theta = \frac{\frac{\lambda_2^2}{n_{o2}^2}\left(\frac{n_{o1}}{\lambda_1}-\frac{n_{o3}}{\lambda_3}\right)^2 - 1}{1-\frac{\lambda_2^2}{n_{z2}^2}\left(\frac{n_{o1}}{\lambda_1}-\frac{n_{o3}}{\lambda_3}\right)^2}$$
***o* ↔ *e* + *e* (positive uniaxial)†**	
$$\frac{n_{o1}}{\lambda_1} = \frac{n_{o2}}{\lambda_2\sqrt{1-\left(1-\frac{n_{o2}^2}{n_{z2}^2}\right)\sin^2\theta}} + \frac{n_{o3}}{\lambda_3\sqrt{1-\left(1-\frac{n_{o3}^2}{n_{z3}^2}\right)\sin^2\theta}}$$	
***e* ↔ *o* + *e* (negative uniaxial)†**	
$$\frac{n_{o1}}{\lambda_1\sqrt{1-\left(1-\frac{n_{o1}^2}{n_{z1}^2}\right)\sin^2\theta}} = \frac{n_{o2}}{\lambda_2} + \frac{n_{o3}}{\lambda_3\sqrt{1-\left(1-\frac{n_{o3}^2}{n_{z3}^2}\right)\sin^2\theta}}$$	
***e* ↔ *e* + *o* (negative uniaxial)†**	
$$\frac{n_{o1}}{\lambda_1\sqrt{1-\left(1-\frac{n_{o1}^2}{n_{z1}^2}\right)\sin^2\theta}} = \frac{n_{o2}}{\lambda_2\sqrt{1-\left(1-\frac{n_{o2}^2}{n_{z2}^2}\right)\sin^2\theta}} + \frac{n_{o3}}{\lambda_3}$$	

†Solved graphically or by using a root-finding algorithm.