2.2 Photon Noise Limitations of Thermal Detectors[1]

Because thermal detectors are sensitive to the absorbed radiant power intensity, whereas photon detectors respond to the rate of photon absorption, the analysis of the photon noise limitations of the two types differ. The photon noise limitation of thermal detectors is imposed by fluctuations in the absorbed power, because of the quasi-random arrival of the photons, whereas photon detectors are limited by fluctuations in the rate of photon absorption. Consider first the photon noise-limited thermal detector.

Assume a thermal detector that is coupled to its environment by radiation interchange alone, that is, there is no energy interchange through the mechanisms of convection or conduction. Assume that the detector is at a temperature T_1, surrounded by a uniform environment at a temperature T_2, and that it has an emissivity η that is independent of T_1 and of wavelength. The noise spectrum of the power emitted by a radiating body in the optical frequency interval $d\upsilon\nu$ is

$$dP_p = 2Ah\upsilon M(\upsilon,T)\frac{\exp(h\upsilon/kT)d\upsilon}{[\exp(h\upsilon/kT)-1]}. \tag{2.1}$$

$M(\upsilon,T)$ is the power per unit area per unit optical frequency interval emitted by a radiating body.

$$M(\upsilon,T) \equiv \frac{2\pi h\upsilon^3/c_o^{\,2}}{[\exp(h\upsilon/kT)-1]}. \tag{2.2}$$

Also, υ is the optical frequency, h is Planck's constant, c_o is the speed of light, T is the absolute temperature, k is Boltzmann's constant, and A is the emitting area, which is the pixel area A_D multiplied by the fill factor β, i.e., the fraction of A_D which has an emissivity η.

The noise power spectrum $P_p(f)$ represents the mean square deviation from the mean of the radiant power and is given by

$$P_p(f) \equiv \int_0^\infty dP_p = \int_0^\infty 2A_D\beta h\upsilon M(\upsilon,T)\frac{\exp(h\upsilon/kT)}{[\exp(h\upsilon/kT)-1]}d\upsilon. \tag{2.3}$$

Thus $P_p(f)$ is termed the mean square noise power per unit bandwidth of the emitted radiation. The integral is found to be

$$P_p(f) = 8A_D\beta\sigma kT_2^5; \tag{2.4}$$

where the subscript 2 has been added to T to refer to the emitting background. Note that the noise power spectrum is frequency independent or "white." The mean square noise power $\overline{P_p^2}$ in a bandwidth B is therefore

$$\overline{P_p^2} = 8A_D\beta\sigma k T_2^5 B\,. \tag{2.5}$$

Consider now the interaction of this radiation noise with the photon noise-limited thermal detector. $A_D\beta$ in this instance represents the receiving area of the pixel. Assume the detector is sensitive to all possible wavelengths and therefore can detect all the incident noise. Since it has an emissivity η, which is identical with the absorptivity, the received mean square noise power will be

$$\overline{P_p^2} = 8A_D\beta\eta\sigma k T_2^5 B\,. \tag{2.6}$$

However, there will also be a contribution to the noise due to random fluctuations in the radiation power emitted by the pixel. The pixel is itself a radiating body, having an emissivity η and a temperature T_1. The quasi-random emission of photons from it, which carry away heat, is known as photon noise. By the same argument as above, the mean square noise power of the emitted radiation in a bandwidth B will be

$$\overline{P_p^2} = 8A_D\beta\eta\sigma k T_1^5 B\,. \tag{2.7}$$

Thus the total mean square radiation noise power in the bandwidth B is

$$\overline{P_p^2} = 8A_D\beta\eta\sigma k (T_1^5 + T_2^5) B\,. \tag{2.8}$$

In the particular instance in which the detector and the surroundings are in equilibrium and therefore are at the same temperature, the mean square noise power will be

$$\overline{P_p^2} = 16A_D\beta\eta\sigma k T^5 B\,. \tag{2.9}$$

The rms noise power for values of A=1 mm^2, β=1, η=1, T=300 K, and B = 1 Hz is

$$\left(\overline{P_p^2}\right)^{1/2} = 5.55\times10^{-12}\,\mathrm{W}. \tag{2.10}$$

The photon noise-limited detectivity of a photon noise-limited thermal detector will now be determined. The noise equivalent power, the incident radiant power required to give a signal voltage equal to the noise voltage in a specified bandwidth, will be numerically equal to $\left(\overline{P_p^2}\right)^{1/2} / \eta$, since the thermal mechanism will transduce the photon noise equally as well as the radiation signal. Therefore D*, the square root of the detector area per unit noise equivalent power in a 1 Hz bandwidth, will be

$$D^* = \frac{\eta}{[8\eta\sigma k(T_1^5 + T_2^5)]^{1/2}} = \frac{\mathrm{cm(Hz)}^{1/2}}{\mathrm{W}}\,. \tag{2.11}$$

Note that D* is independent of $A_D\beta$, as is to be expected.

In many practical instances the temperature of the background, T_2, will be room temperature, 290 K. For many detectors, such as thermopiles and bolometers, the detector temperature will also be 290 K. Figure 2-1 shows the photon noise-limited detectivity for an ideal thermal detector having an emissivity of unity, operated at 290 K and lower, as a function of detector temperature T_1 and background temperature T_2. Note that Eq. (2.11) is symmetrical in T_1 and T_2. Thus T_1 and T_2 can be interchanged in Fig. 2-1.

It can be seen that the highest possible D* to be expected from a thermal detector operated at room temperature and viewing a background at room temperature is 1.98×10^{10} cm Hz$^{1/2}$/W. Even if the detector or background, not both, were cooled to absolute zero, the detectivity would improve only by the square root of two. This is a basic limitation of all thermal detectors.

Note that Eq. (2.11) and Fig. 2-1 assume that background radiation falls upon the pixel from all directions when the detector and background temperatures are equal, and from the forward hemisphere only when the detector is at cryogenic temperatures. In the latter case, if the field of view is reduced by cold shielding, and the pixel remains background limited, the D* value will depend inversely upon the sine of the half angle of θ where θ is the included angle of the cold shield (see Fig. 2-2).

2.3 Temperature Fluctuation Noise in Thermal Detectors[1]

Another approach to determining the photon noise limited performance of thermal detectors is through the concept of temperature fluctuation noise. A thermal detector in contact with its environment by conduction and radiation exhibits random fluctuations in temperature, known as temperature fluctuation noise, because of the statistical nature of the heat interchange with its surroundings. If the conduction interchange is negligible compared to the

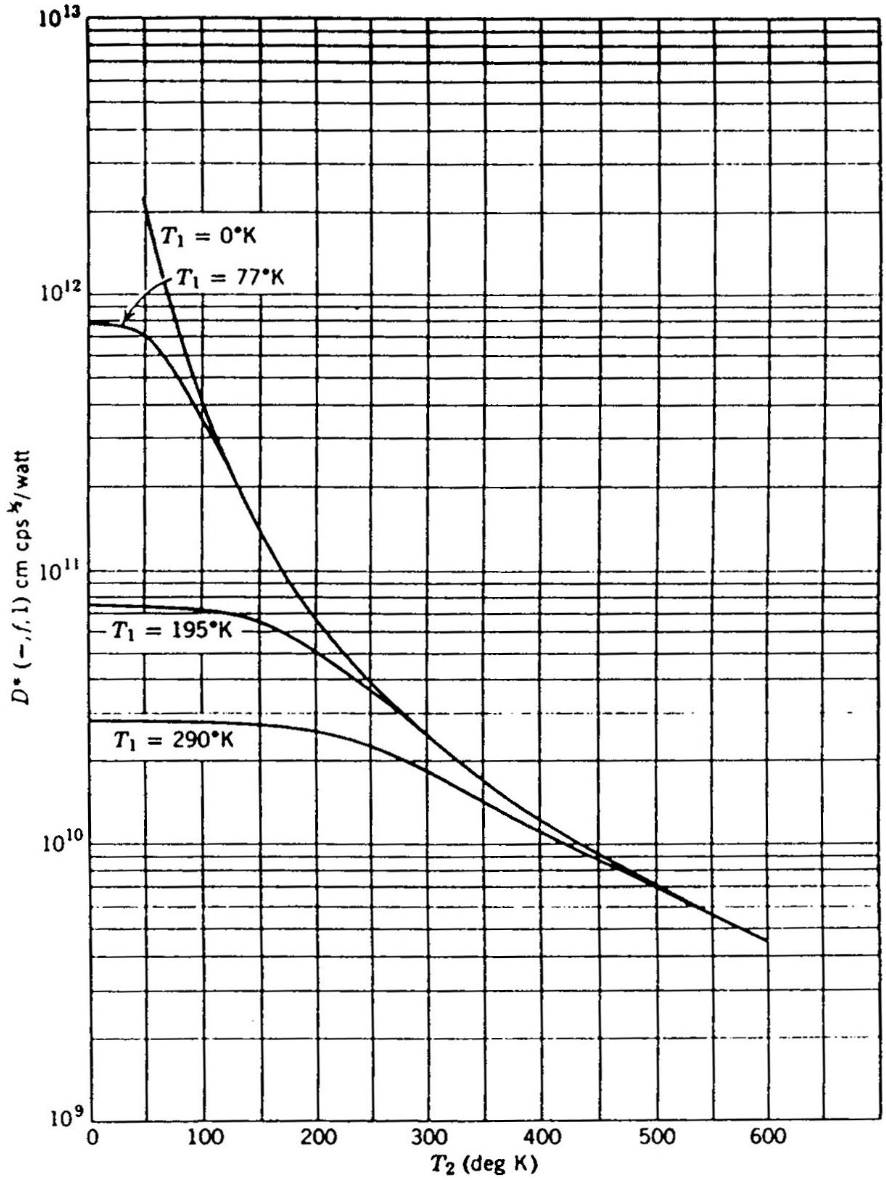

Figure 2-1. Photon noise-limited D* of thermal detectors as a function of detector temperature T_1 and background temperature T_2. Viewing angle of 2π steradians and unit absorptivity. From P.W. Kruse, L.D. McGlauchlin, and R.B. McQuistan.[1]

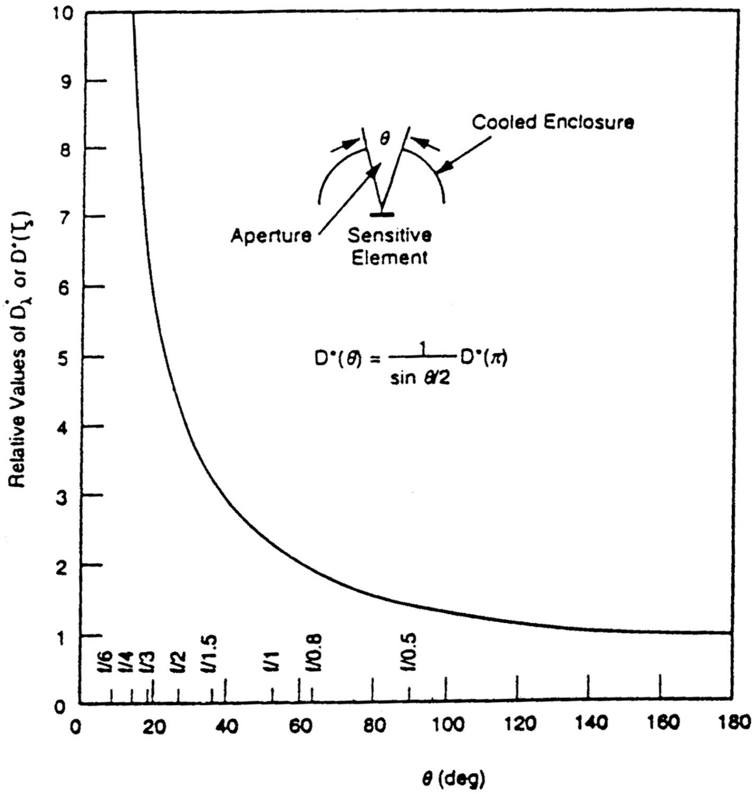

Figure 2-2. Relative increase in background fluctuation noise-limited D^*_λ and $D^*(T)$ for photon and thermal detectors achieved by using cold aperture. From P.W. Kruse, L.D. McGlauchlin, and R.B. McQuistan.[1]

radiation interchange, temperature fluctuation noise would be expected to become identified with background fluctuation noise. The following discussion demonstrates this mathematically. Consider first of all the magnitude of the temperature fluctuations of the detecting material. The material, having a heat capacity C, changes its temperature T by the incremental amount ΔT in response to the energy increment ΔE according to

$$\Delta E = C\,\Delta T. \tag{2.12}$$

The thermodynamic system composed of the material and surroundings possesses many degrees of freedom. Tolman[2] states that the mean square fluctuations in energy $\overline{\Delta E^2}$ of a system having many degrees of freedom is given by

$$\overline{\Delta E^2} = kT^2 C\,; \tag{2.13}$$

where k is Boltzmann's constant. Therefore

$$\overline{\Delta T^2} = \frac{kT^2}{C}. \tag{2.14}$$

Einstein[3] showed that C in this case is the harmonic mean of the heat capacities of the material and surroundings.

$$C = \frac{C_A C_B}{C_A + C_B}\,; \tag{2.15}$$

where C_A and C_B are the heat capacities of the body and surroundings, respectively. If the surroundings have a much greater heat capacity than the body, which is the usual situation, the harmonic mean heat capacity becomes that of the body.

Next consider the spectral content of the fluctuations. Let the detecting material be at an incremental temperature difference ΔT above its surroundings. Heat will flow from the material to the surroundings at a rate proportional to ΔT, the proportionality constant being the heat transfer coefficient G between the material and the surroundings. The heat transfer equation is

$$\frac{d(\Delta E)}{dt} = G\Delta T\,; \tag{2.16}$$

where $d(\Delta E)/dt$ is the rate of flow of heat. However, Eq. (2.12) shows that the rate of flow of heat can also be expressed as

$$\frac{d(\Delta E)}{dt} = C\frac{d(\Delta T)}{dt}\,. \tag{2.17}$$

If the material is at a higher temperature than its surroundings, heat flows from the material to the surroundings and $d(\Delta E)/dt$ is negative. Equating Eqs. (2.16) and (2.17), a differential equation describing the heat transfer is found to be

$$-C\frac{d(\Delta T)}{dt} = G\Delta T. \tag{2.18}$$

The solution of this is