Field Guide to

Probability, Random Processes, and Random Data Analysis

Larry C. Andrews
Ronald L. Phillips

SPIE Field Guides
Volume FG22

John E. Greivenkamp, Series Editor

SPIE
PRESS

Bellingham, Washington USA

Library of Congress Cataloging-in-Publication Data

Andrews, Larry C.
Field guide to probability, random processes, and random data analysis / Larry C. Andrews, Ronald L. Phillips.
p. cm. – (Field guide series)
Includes bibliographical references and index.
ISBN 978-0-8194-8701-8
1. Mathematical analysis. 2. Probabilities. 3. Random data (Statistics) I. Phillips, Ronald L. II. Title.
QA300.A5583 2012
519.2–dc23

2011051386

Published by

SPIE
P.O. Box 10
Bellingham, Washington 98227-0010 USA
Phone: +1.360.676.3290
Fax: +1.360.647.1445
Email: books@spie.org
Web: http://spie.org

Printed in the United States of America.

First Printing

Introduction to the Series

Welcome to the *SPIE Field Guides*—a series of publications written directly for the practicing engineer or scientist. Many textbooks and professional reference books cover optical principles and techniques in depth. The aim of the *SPIE Field Guides* is to distill this information, providing readers with a handy desk or briefcase reference that provides basic, essential information about optical principles, techniques, or phenomena, including definitions and descriptions, key equations, illustrations, application examples, design considerations, and additional resources. A significant effort will be made to provide a consistent notation and style between volumes in the series.

Each *SPIE Field Guide* addresses a major field of optical science and technology. The concept of these Field Guides is a format-intensive presentation based on figures and equations supplemented by concise explanations. In most cases, this modular approach places a single topic on a page, and provides full coverage of that topic on that page. Highlights, insights, and rules of thumb are displayed in sidebars to the main text. The appendices at the end of each Field Guide provide additional information such as related material outside the main scope of the volume, key mathematical relationships, and alternative methods. While complete in their coverage, the concise presentation may not be appropriate for those new to the field.

The *SPIE Field Guides* are intended to be living documents. The modular page-based presentation format allows them to be easily updated and expanded. We are interested in your suggestions for new Field Guide topics as well as what material should be added to an individual volume to make these Field Guides more useful to you. Please contact us at **fieldguides@SPIE.org**.

John E. Greivenkamp, *Series Editor*
College of Optical Sciences
The University of Arizona

The Field Guide Series

Keep information at your fingertips with all of the titles in the Field Guide Series:

Field Guide to

Adaptive Optics, Robert Tyson & Benjamin Frazier

Atmospheric Optics, Larry Andrews

Binoculars and Scopes, Paul Yoder, Jr. & Daniel Vukobratovich

Diffractive Optics, Yakov G. Soskind

Geometrical Optics, John Greivenkamp

Illumination, Angelo Arecchi, Tahar Messadi, & John Koshel

Infrared Systems, Detectors, and FPAs, *Second Edition*, Arnold Daniels

Interferometric Optical Testing, Eric Goodwin & Jim Wyant

Laser Pulse Generation, Rüdiger Paschotta

Lasers, Rüdiger Paschotta

Microscopy, Tomasz Tkaczyk

Optical Fabrication, Ray Williamson

Optical Fiber Technology, Rüdiger Paschotta

Optical Lithography, Chris Mack

Optical Thin Films, Ronald Willey

Polarization, Edward Collett

Radiometry, Barbara Grant

Special Functions for Engineers, Larry Andrews

Spectroscopy, David Ball

Visual and Ophthalmic Optics, Jim Schwiegerling

Field Guide to Probability, Random Processes, and Random Data Analysis

Developed in basic courses in engineering and science, mathematical theory usually involves deterministic phenomena. Such is the case for solving a differential equation that describes a linear system where both input and output are deterministic quantities. In practice, however, the input to a linear system, such as imaging or radar systems, can contain a "random" quantity that yields uncertainty about the output. Such systems must be treated by probabilistic rather than deterministic methods. For this reason, probability theory and random-process theory have become indispensable tools in the mathematical analysis of these kinds of engineering systems.

Topics included in this *Field Guide* are basic probability theory, random processes, random fields, and random data analysis. The analysis of random data is less well known than the other topics, particularly some of the tests for stationarity, periodicity, and normality.

Much of the material is condensed from the authors' earlier text *Mathematical Techniques for Engineers and Scientists* (SPIE Press, 2003). As is the case for other volumes in this series, it is assumed that the reader has some basic knowledge of the subject.

Larry C. Andrews
Professor Emeritus
Townes Laser Institute
CREOL College of Optics
University of Central Florida

Ronald L. Phillips
Professor Emeritus
Townes Laser Institute
CREOL College of Optics
University of Central Florida

Table of Contents

Table of Contents

Table of Contents

Glossary of Symbols and Notation

$\mathbf{a}, \mathbf{x}, \mathbf{u}$, etc.	Random variable, process, or field
$B_{\mathbf{u}}(R)$	Autocovariance or covariance function of random field
$C_{\mathbf{x}}(\tau)$	Autocovariance or covariance function of random process
$C_{\mathbf{xy}}(\tau)$	Cross-covariance function
CDF	Cumulative distribution function
Cov	Covariance
$D_{\mathbf{x}}(\tau)$	Structure function
$E[.]$	Expectation operator
$E[g(\mathbf{x})\|A]$	Conditional expectation operator
$f_{\mathbf{x}}(x), f_{\mathbf{x}}(x,t)$	Probability density function
$f_{\mathbf{x}}(x\|A)$	Conditional probability density
$F_{\mathbf{x}}(x), F_{\mathbf{x}}(x,t)$	Cumulative distribution function
$F_{\mathbf{x}}(x\|A)$	Conditional cumulative distribution function
${}_pF_q$	Generalized hypergeometric function
$h(t)$	Impulse response function
$H(\omega)$	Transfer function
$I_p(x)$	Modified Bessel function of the first kind
$J_p(x)$	Bessel function of the first kind
$K_p(x)$	Modified Bessel function of the second kind
m, $m(t)$	Mean value
m_k	k'th standard statistical moment
$n!$	Factorial function
PDF	Probability density function
Pr	Probability
$Pr(B\|A)$	Conditional probability
PSD	Power spectral density
RV	Random variable
$R_{\mathbf{x}}(\tau)$	Autocorrelation or correlation function
$R_{\mathbf{xy}}(\tau)$	Cross-correlation function
$\Re_{\mathbf{x}}(\tau)$	Long-time-average correlation function
$S_{\mathbf{x}}(\omega), S_{\mathbf{u}}(\kappa)$	Power spectral density function
$U(x-a)$	Unit step function

Glossary of Symbols and Notation

Var	Variance
Var$[\mathbf{x}\|A]$	Conditional variance
$\overline{\mathbf{x}(t)}$	Time average
z^*	Complex conjugate of z
$\gamma(c,x)$	Incomplete gamma function
$\Gamma(x)$	Gamma function
$\delta(x-a)$	Dirac delta function (impulse function)
μ_k	k'th central statistical moment
$\hat{\mu}(t)$	Estimator of mean value
$\sigma^2, \sigma_{\mathbf{x}}^2$	Variance
τ	Time difference $t_2 - t_1$
$\Phi_{\mathbf{x}}(s)$	Characteristic function
$\|\|$	Absolute value
$\in$	Belonging to
$\binom{a}{n}$	Binomial coefficient
$\langle\,\rangle$	Ensemble average
$\{\}$	Event
$\cap$	Intersection