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Abstract. Microgrid polarization cameras have historically experienced lower performance than is typical for monolithic polarization systems. Specifically, their polarizer elements have had a lower extinction ratio and a larger orientation error. We show how to use the calibrated parameters of a nonideal polarizer to modify the polarization measurement model, effectively allowing one to generate high-performance measurements from low-performance elements. We demonstrate the effectiveness of this approach on a commercial polarization camera and estimate the signal-to-noise ratio penalty for using non-ideal polarizers in this camera as being 1.25x versus a system using ideal polarizers. © The Authors. Published by SPIE under a Creative Commons Attribution 4.0 Unported License. Distribution or reproduction of this work in whole or in part requires full attribution of the original publication, including its DOI. [DOI: 10.1117/1.OE.58.8.080501]

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A polarizer can be used to estimate the orientation and degree of linear polarization of light by rotating the polarizer to the four angles: 0 deg, 45 deg, 90 deg, and 135 deg and measuring the light intensity transmitted at each angle. In a polarization camera, this can be done in a single snapshot by combining the measurements of four adjacent pixels, each of which have micropolarizers oriented at each of the four orientation angles (Fig. 1). For incoherent light, we can use the Stokes vector \( s = (s_0, s_1, s_2, s_3)^T \) to represent the input polarization state and the Mueller calculus to model the four intensities \( I_0 \) measured at the detector:

\[
I_0 = \frac{1}{2} (s_0 + s_1) + n_0, \\
I_{45} = \frac{1}{2} (s_0 + s_2) + n_{45}, \\
I_{90} = \frac{1}{2} (s_0 - s_1) + n_90, \\
I_{135} = \frac{1}{2} (s_0 - s_2) + n_{135}.
\]  

(1)

for random noise values \( n_i \). In the analysis to follow, we will assume all of the noise terms to have zero mean, i.e., \( \langle I_i + n_i \rangle = I_i \) and \( \langle n_i \rangle = 0 \). This causes no difficulties for Poisson-distributed noise, since our definition of the signal \( I \) and noise \( n \) results in the mean value of the Poisson-distributed variable to be incorporated into \( I \) while \( n \) retains the zero-mean stochastic portion.

The four equations of (1) have the matrix form

\[
\begin{pmatrix}
I_0 \\
I_{45} \\
I_{90} \\
I_{135}
\end{pmatrix}
= \frac{1}{2}
\begin{pmatrix}
1 & 1 & 0 & 0 \\
1 & 0 & 1 & 0 \\
1 & -1 & 0 & 0 \\
1 & 0 & -1 & 0
\end{pmatrix}
\begin{pmatrix}
s_0 \\
s_1 \\
s_2 \\
s_3
\end{pmatrix}
+ \begin{pmatrix}
n_0 \\
n_{45} \\
n_90 \\
n_{135}
\end{pmatrix}
\]

or \( I = Ws + n \), where \( W \) is the measurement matrix. By inspection, we can obtain the input linear Stokes vector elements as

\[
\begin{align*}
\hat{s}_0 &= \frac{1}{2} (I_0 + I_{45} + I_{90} + I_{135}), \\
\hat{s}_1 &= I_0 - I_{90}, \\
\hat{s}_2 &= I_{45} - I_{135}.
\end{align*}
\]

(3)

or

\[
\begin{pmatrix}
\hat{s}_0 \\
\hat{s}_1 \\
\hat{s}_2
\end{pmatrix}
= \frac{1}{2}
\begin{pmatrix}
1 & 1 & 1 & 1 \\
2 & 0 & -2 & 0 \\
0 & 2 & 0 & -2
\end{pmatrix}
\begin{pmatrix}
I_0 \\
I_{45} \\
I_{90} \\
I_{135}
\end{pmatrix}.
\]

(4)

i.e., \( s = A \hat{s} \), where \( A \) is the analysis matrix. To evaluate the performance of a polarization measurement, we can use the measurement and analysis matrix elements to calculate the Stokes vector covariance matrix \( K \) as

\[
K_{ij} = \frac{3}{4} \sum_{k=0}^{3} \sum_{\ell=0}^{3} A_{ik} A_{j\ell} W_{k\ell}
+ \frac{3}{4} \sum_{\ell=0}^{3} A_{i\ell} A_{j\ell}
\]

Poisson noise part

Gaussian noise part

where \( s_k \) is the \( k \)’th Stokes vector element in units of photons and \( n_d \) is the detector noise variance, also in units of photoelectrons. For an ideal polarizer, this set of four measurements produces variances in the Stokes vector components as

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The equally weighted variance (EWV) is often used to summarize the measurement performance, such that in this case EWV = \( \frac{1}{2} s_0 + v_d \).

A nonideal linear polarizer is a diattenuator defined by its diattenuation \( A \), its orientation \( \alpha \), and its efficiency \( \eta \) (the average transmission of the diattenuator for all input polarization angles):\(^4\)

\[
M_{id} = \eta \begin{pmatrix} 1 & Dc & Ds & 0 \\ Dc & 1 - Ds^2 & \frac{1}{2\eta} Dcs & 0 \\ Ds & \frac{1}{2\eta} Dcs & 1 - Dc^2 & 0 \\ 0 & 0 & 0 & 1 - D \end{pmatrix},
\]

for \( c = \cos(2\alpha) \) and \( s = \sin(2\alpha) \). An ideal polarizer has \( D = 1 \), \( \eta = 1 \), and orientation \( \alpha \). The polarizer’s extinction ratio \( X \) is derived from the diattenuation as \( X = (1 + D)/(1 - D) \).

Modeling each pixel of a polarization camera as a nonideal polarizer, we can calibrate the diattenuation \( D_n \) and orientation \( \alpha_n \) at each pixel \( n \), then use the Mueller calculus to estimate the polarization state by combining measurements \( I_i \) from each set of four neighboring pixels. This produces the measurement model\(^5\)

\[
\begin{pmatrix} I_0 \\ I_1 \\ I_2 \\ I_3 \end{pmatrix} = \frac{1}{2} \begin{pmatrix} 1 & D_0 \cos(2\alpha_0) & D_0 \sin(2\alpha_0) & 0 \\ 1 & D_1 \cos(2\alpha_1) & D_1 \sin(2\alpha_1) & 0 \\ 1 & D_2 \cos(2\alpha_2) & D_2 \sin(2\alpha_2) & 0 \\ 1 & D_3 \cos(2\alpha_3) & D_3 \sin(2\alpha_3) & 0 \end{pmatrix} \begin{pmatrix} s_0 \\ s_1 \\ s_2 \\ s \end{pmatrix}.
\]

The polarizer efficiency \( \eta \) does not appear here because it typically cannot be separated from the pixel’s internal quantum efficiency \( q_{int} \)—the two appear together as the single factor \( q_{ext} = \eta q_{int} \) as the "external" quantum efficiency. Because we do not calibrate the camera’s quantum efficiency here, the equations below assume \( q_{ext} = 1 \), so that the Stokes parameters are represented in units of photoelectrons rather than photons.

The general analytical form of \( A \) is quite complex, but if we assume that the orientation errors are small then we can use a Taylor expansion about their nominal values:

\[
\begin{align*}
\cos(2\alpha_0) &= \cos(2\epsilon_0) \rightarrow 1, \\
\cos(2\alpha_1) &= \cos(90^\circ + 2\epsilon_1) \rightarrow -2\epsilon_1, \\
\cos(2\alpha_2) &= \cos(180^\circ + 2\epsilon_2) \rightarrow -1, \\
\cos(2\alpha_3) &= \cos(270^\circ + 2\epsilon_3) \rightarrow 2\epsilon_3,
\end{align*}
\]

and

\[
\begin{align*}
\sin(2\alpha_0) &= \sin(2\epsilon_0) \rightarrow 2\epsilon_0, \\
\sin(2\alpha_1) &= \sin(90^\circ + 2\epsilon_1) \rightarrow 1, \\
\sin(2\alpha_2) &= \sin(180^\circ + 2\epsilon_2) \rightarrow -2\epsilon_2, \\
\sin(2\alpha_3) &= \sin(270^\circ + 2\epsilon_3) \rightarrow -1,
\end{align*}
\]

where \( \epsilon_\alpha \) represents the angular error about the nominal orientation. Likewise, each diattenuation can be represented as \( D_i = 1 - \Delta_i \), where the diattenuation errors \( \Delta_i \) are assumed to be small. For this first-order approximation regime, the analysis matrix \( A \) in the Stokes vector estimation:

\[
\begin{pmatrix} s_0' \\ s_1' \\ s_2' \\ s \end{pmatrix} = \frac{1}{4} \begin{pmatrix} A_{00} & A_{01} & A_{02} & A_{03} \\ A_{10} & A_{11} & A_{12} & A_{13} \\ A_{20} & A_{21} & A_{22} & A_{23} \\ A & A & A & A \end{pmatrix} \begin{pmatrix} I_0 \\ I_1 \\ I_2 \\ I_3 \end{pmatrix}
\]

simplifies to

\[
\begin{align*}
A_{00} &= 2 + \Delta_0 - \Delta_2 + 2\epsilon_1 - 2\epsilon_3, \\
A_{01} &= 2 + \Delta_1 - \Delta_3 - 2\epsilon_0 + 2\epsilon_2, \\
A_{02} &= 2 - \Delta_0 + \Delta_2 - 2\epsilon_1 + 2\epsilon_3, \\
A_{03} &= 2 - \Delta_1 + \Delta_3 + 2\epsilon_0 - 2\epsilon_2, \\
A_{10} &= 4 + \Delta_0 + 3\Delta_2 + 2\epsilon_1 - 2\epsilon_3, \\
A_{11} &= \Delta_0 - \Delta_2 - 4\epsilon_0 + 2\epsilon_1 + 4\epsilon_2 - 2\epsilon_3, \\
A_{12} &= -4 - 3\Delta_0 - \Delta_2 - 2\epsilon_1 - 2\epsilon_3, \\
A_{13} &= \Delta_0 - \Delta_2 + 4\epsilon_0 - 2\epsilon_1 + 4\epsilon_2 + 2\epsilon_3, \\
A_{20} &= \Delta_0 - \Delta_3 - 2\epsilon_0 + 4\epsilon_1 - 2\epsilon_2 + 4\epsilon_3, \\
A_{21} &= 4 + \Delta_1 + 3\Delta_3 - 2\epsilon_0 + 2\epsilon_2, \\
A_{22} &= \Delta_1 - \Delta_3 + 2\epsilon_0 - 4\epsilon_1 - 2\epsilon_2 - 4\epsilon_3, \\
A_{23} &= -4 - 3\Delta_1 - \Delta_3 - 2\epsilon_0 + 2\epsilon_2.
\end{align*}
\]

Here, \( s_i' \) indicates the calibration-corrected Stokes estimate, while \( s_i \) without a prime indicates the estimate assuming an ideal polarizer. In Eqs. (9) and (10), note that it is generally not useful to define \( s_0 \) as anything but zero, since the system reference axis is a degree of freedom that we have to choose in most situations. In this case, we can set \( \epsilon_0 = 0 \).

To demonstrate this technique, we use a commercially available polarization camera to measure the Stokes vector elements of the light transmitted through a high-extinction-ratio Glan–Thompson polarizer as the polarizer is rotated through a set of angles from 0 deg to 180 deg. The polarization camera is a photonic lattice PI-110, having

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\( \text{Fig. 1 A polarization camera showing a 2} \times \text{2 pixels group from the detector array and their polarization filter orientations.} \)
increases linearly with \( \Delta_0 \) and \( \Delta_2 \), and \( \text{var}(\tilde{s}_i) \) with \( \Delta_1 \) and \( \Delta_3 \), respectively. The Stokes parameters are often expressed in normalized form \( \tilde{s}_i = s_i / s_0 \), for which the expressions in (11) need to be modified. The normalized parameter variances are related to their unnormalized counterparts by

\[
\text{var}(\tilde{s}_i) = \frac{1}{s_0^4} \text{var}(s_i) + \frac{\langle s_i'^2 \rangle}{s_0^4} \text{var}(s'_0),
\]

(12)

which together with Eq. (11) gives

\[
\begin{align*}
\text{var}(\tilde{s}_1) &= \frac{v_d}{s_0^2} (2[1 + \Delta_0 + \Delta_2] + \tilde{s}_1^2), \\
\text{var}(\tilde{s}_2) &= \frac{v_d}{s_0^2} (2[1 + \Delta_1 + \Delta_3] + \tilde{s}_2^2),
\end{align*}
\]

(13)

and where we have replaced the corrected parameter means with the true parameter values: \( \langle s'_0 \rangle \rightarrow s_0 \), etc.

To see how these variance equations behave quantitatively, we can look at an example group of 4 pixels on the polarization camera. For this group, we obtain calibrated diattenuations of

\[
D_0 = 0.8277, \quad D_1 = 0.7435, \quad D_2 = 0.8133, \quad D_3 = 0.7338,
\]

and calibrated orientation angles of

\[
\alpha_0 = 0 \text{ deg}, \quad \alpha_1 = 49.650 \text{ deg}, \quad \alpha_2 = 88.694 \text{ deg}, \quad \alpha_3 = 134.399 \text{ deg},
\]

so that the differences from the nominal angles are 0 deg, 4.650 deg, -1.306 deg, and -0.601 deg. Converting the angular errors to radians for the \( \epsilon_i \) values, we obtain the error parameters:

\[
\begin{align*}
\Delta_0 &= 0.1723, \quad \Delta_1 = 0.2565, \quad \Delta_2 = 0.1867, \quad \Delta_3 = 0.2662, \\
\epsilon_0 &= 0, \quad \epsilon_1 = 0.0811, \quad \epsilon_2 = -0.0228, \quad \epsilon_3 = -0.0105.
\end{align*}
\]

(14)

With these parameters, the variances (11) in the detector-limited noise regime (uniform Gaussian noise) become, via Eq. (5),

\[
\begin{align*}
\text{var}(\tilde{s}'_0) &= 1.00 v_d, \quad \text{var}(\tilde{s}'_1) = 3.23 v_d, \quad \text{var}(\tilde{s}'_2) = 3.01 v_d.
\end{align*}
\]

(15)

for detector noise variance \( v_d \). In comparison to Eq. (6), we see that the corrected nonideal measurement for this pixel group suffers an increase in the variances for measuring \( s_1 \) and \( s_2 \) by factors of 1.62 and 1.51, respectively, while the variance for \( s_0 \) is basically unaffected. These correspond to a loss in signal-to-noise ratio (SNR) by factors of 1.27 in \( s_1 \) and 1.23 in \( s_2 \).

Using the same Eqs. (2), (5), and (10), we can obtain similar results for the Poisson noise regime, for which the variance equations become
is that the analytic dependence of the Stokes parameter variance is expressed here in terms of the calibration parameters directly, allowing us to see more clearly the effect of misalignment or low diattenuation on the measurements.

In Fig. 2, we can see that the mean values of the corrected curves come close to the ideal behavior of a cosine curve oscillating between +1 and −1. The shaded regions surrounding each curve describe the values within one standard deviation about the mean, where we can expect to find the data most of the time. The noise fluctuations result primarily from the shot noise of the measurements, but are also influenced by any error in the calibration parameters. As we can expect for any linear estimation procedure, as the estimate approaches the ideal cosine curve, noise fluctuations cause some of the data at the peaks and valleys to extend beyond the physically valid region between +1 and −1.

From these results, we can see that most polarimeters can tolerate a surprisingly low diattenuation if calibrated well. As often is the case for indirect measurements, it is the accuracy with which one can calibrate the system measurement model that limits the measurement performance more than the inherent accuracy of the components themselves. If we work with low-extinction-ratio polarizers, we can still achieve high-accuracy measurements, but only if we take extreme care in the calibration and in ensuring the accuracy of our reconstruction.

For example, we should keep in mind that the diattenuation value of a pixel is a spectrally dependent quantity, so that the measurement spectrum must be similar to the spectrum used during calibration in order for the results to be accurate.

References