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ABSTRACT

The results of an investigation of polymethylmethacrylate deformation due to applied high-load utilizing its birefringence are presented in the paper. The sample under the test was placed between two plane polarizers, illuminated by white light and gradually stressed using a machine for fatigue testing of materials. The spectra of transmitted light were measured as function of load and deformation using optical fiber spectral analyzers for visible (VIS) and near infrared (NIR) spectral ranges. The measured spectra were normalized and light intensities for selected wavelengths as function of load and deformation were compared with the dependences according to models describing the stress-induced birefringence of PMMA as function of load and deformation, respectively. A good match between the dependences resulting from the measured data and those calculated according to models implies the models can be used for calculating calibrations curves of PMMA-based sensors of load and/or deformation.

Keywords: PMMA, deformation, stress, plane polariscope, stress-induced birefringence

1. INTRODUCTION

Often, solid-state transparent polymeric materials exhibit birefringence. In many applications based on their mechanical properties only, the birefringence occurring in these polymers plays no role. However, if a material shows birefringence and its variations due to any external conditions, it may be considered interesting for sensor applications. Mainly, when the variations of the birefringence due to external fields are very well defined and the change of birefringence can simply be manifested by change of some well detectable quantity, such as change of specimen color or intensity of light that passed through the birefringent specimen. One of the commonly used transparent polymeric materials showing the birefringence and its changes due to mechanical stress is polymethylmethacrylate (PMMA). Due to its transparency in VIS spectral range and good mechanical properties, it is often used in roofing and waterproofing applications, in automotive and transportation, architecture, electronics and health, for production of lenses, optical fibers, polymer fiber lasers and photonic structures for sensors and display applications. The recent investigations of the influence of the applied load on the strength of the photoelastic birefringence of PMMA in visible and near-infrared spectral ranges show the potential of the effect for sensor applications. In the paper we discuss the potential of PMMA for sensor applications based on measuring the applied loads up to thousands Newtons and deformations up to several tenths of millimeter.

2. PHYSICAL PRINCIPLE OF THE SENSOR ELEMENT

It is known that a monochromatic light wave propagating in a birefringent medium can be regarded as a sum of two mutually orthogonal linearly polarized eigenwaves. Due to different velocities they possess, these waves are assigned different refractive indices called ordinary \( n_o \) and extraordinary \( n_e \). As the waves propagate inside the medium, the path difference between them arises. At the end of the medium with thickness \( d \) the total phase shift \( \Gamma \) between eigenwaves depends on the distance \( d \) the waves travelled, the wavelength \( \lambda \), and the difference between their refractive indices, \( n_o - n_e \) also called birefringence. Thus, the phase shift \( \Gamma \) depends on birefringence and defines the polarization state of the wave emerging from the medium.

\[
\Gamma = \frac{2\pi}{\lambda} \cdot d \cdot (n_o - n_e).
\]

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In practice, the polarization state of the wave, thus the birefringence of a material, can be simply analyzed using polarizing elements and an unpolarized light source. Placing a birefringent medium between two crossed polarizers and illuminating the system by white light, the intensity $I$ of light transmitted through the system will vary with wavelength of light, thickness $d$ of the medium and birefringence of the medium as well, according to

$$I \propto I_0 \cdot \left[ \sin \left( \frac{\Gamma}{2} \right) \right]^2,$$

where $I_0$ denotes the intensity of light incident on the medium. Moreover, in case the birefringence of the medium varies due to force applied perpendicularly to the direction of light propagation (Fig. 1) as a result one will observe changes in spectral distribution of light intensity. In principle, measurement of the intensity of light as function of loading or deformation of the birefringent specimen can be used as a basis for quantifying the parameters such as load, deformation, displacement, weight or pressure applied on the sensor element.

![Experimental set-up used for investigation of the PMMA-based sensor element response to the applied load.](image)

Figure 1. Experimental set-up used for investigation of the PMMA-based sensor element response to the applied load. Arrows inside the polarizing elements - Polarizer and Analyzer denote the orientation of the planes of polarization. OSA – VIS/NIR optical spectrum analyzer.

3. EXPERIMENT

The tested samples of the sensor element were cut from commercially available plates of clear (colorless) PMMA. The dimensions $x$, $y$ and $z$ of the prepared PMMA blocks were 82.95mm × 19.92mm × 9.57mm, respectively. As a source of white light a halogen lamp SLS201/M (Thorlabs) was used. The investigation was performed in VIS and NIR spectral ranges using plane polarizers for wavelengths from 400 to 800nm and from 900 to 1800nm, respectively. Light that passed through the polarizer-sample-analyzer was collected by an optical fiber connected to VIS and NIR optical spectral analyzers HR2000® and NIRQuest 512 (Ocean Optics), respectively. In order to gain a sufficient optical signal a fiber with 400μm diameter and numerical aperture 0.39 was used for collecting light. The influence of the load on the birefringence of PMMA samples indicated by change of spectral distribution of intensity of transmitted light was investigated by a machine for fatigue testing of materials Vibrophore Amsler 150 HFP 5100 (Zwick/Roell). The measurement was performed at room temperature.

3.1 Load versus deformation

The PMMA sample was put on a metal pad placed on a lower head of the loading machine and a simple test was performed in order to prove the estimated range of loads inducing elastic deformation. The load applied onto the sample by the upper head of the machine gradually increased from 0 to 20kN and we measured the displacement of the upper loading head from its initial position during the sample loading. The displacement was measured by a gauge Imeko EDK 93. The accuracy of the displacement measurement was 0.01 μm while the probe was able to travel a total distance of 1.2mm. After reaching the maximal estimated value of load the load was gradually decreased while the displacement of the upper head of the machine was still measured.
The dependence of the deformation of the PMMA sample represented by strain $\varepsilon$ as function of increasing/decreasing load is shown in Fig. 2. Since the deformation is due to compression of the sample, the strain $\varepsilon$ is negative.

Comparison between deformations measured during sample loading and unloading shows that no significant plastic deformation occurs within the used range of loads.

3.2 Load versus spectral distribution of light intensity

The determination of the range of loads inducing elastic deformations was followed by measuring the spectra of light transmitted through the sample placed between crossed plane polarizers. The load was gradually applied on the sample and spectra of transmitted light were recorded for every load applied. The normalized dependences of the spectral distribution of intensity of light on the load measured in VIS and NIR spectral ranges are shown in Fig. 3.
The dependences shown in Fig. 3 are displayed in wavelength ranges that match the region of high polarization efficiency of the used polarizers, i.e. from 480 to 740nm and from 900 to 1780nm for VIS and NIR spectral ranges, respectively. These normalized dependences were used for further analysis.

4. ANALYSIS OF MEASURED DATA

In order to quantify the effect of the applied mechanical stress on the intensity of light transmitted through the PMMA sample we analyzed the normalized dependences of light intensity on the load as well as on the deformation for selected wavelengths.

4.1 Intensity of light as function of load

The change of birefringence of a material due to mechanical stress or strain is known as photoelasticity and can be quantified according to stress-optic law

$$\Delta n = \Delta n_0 + C \cdot \Delta \sigma,$$

where $\Delta n_0$ is the birefringence of the stress-free material (which can also be zero), $C$ is the so-called relative stress-optic coefficient and $\Delta \sigma$ is the difference between principal stresses occurring inside the material in plane perpendicular to the direction of propagation of light. Substituting Eq. (3) into Eq. (1) and Eq. (2) we get for the intensity of light

$$I \propto I_0 \cdot \sin \left( \frac{\pi \cdot d \cdot \left( \Delta n_0 + C \cdot \frac{F}{S} \right)}{\lambda} \right)^2,$$

where the principal stresses difference $\Delta \sigma$ is expressed by the ratio $F/S$ of the applied force $F$ and the area $S$ of the loaded upper side of the sample. The dependences of normalized intensity on load, both measured and calculated according to Eq. (4), are for selected wavelengths in VIS and NIR ranges shown in Fig. 4. The fitting was realized by a code written in Mathcad 14 using Levenberg–Marquardt method with $\Delta n_0$ and $C$ being the fitted parameters.

Figure 4. Measured and calculated normalized spectral distribution of light intensity for selected wavelengths in VIS (a) and NIR (b) spectral ranges as function of load.
As can be seen in Fig. 4, a good match between calculated dependences and the measured data is observed in the whole range of loads. This predictable behavior of intensity of light with respect to the applied load represented by the model according to Eq. (4) can be utilized for setting a calibration curve of the PMMA sensor element. In VIS spectral range the ambiguity of the light intensity dependence on the load limits the utilization of visible light only for sensing the lower loads. On the other hand, the dependences in Fig. 4b show that light from NIR spectral range can preferably be used for sensing higher loads.

4.2 Intensity of light as function of deformation

The measured dependence of intensity of light transmitted through the PMMA sample on the load as well as the dependence of the strain on the load can be combined and the dependence of the intensity of transmitted light on the strain can be plotted. In order to be able to calculate the intensity of light for any wavelength as function of the strain one needs to know the dependence of the birefringence on the deformation expressed by the strain \( \varepsilon \). The connection between strain and birefringence comes from the stress-optic law expressed by Eq. (3) and the relationship between stress and strain, as well. Based on the measured load, dimensions of the sample and the displacement of the upper loading head from its initial position the engineering stress-engineering strain dependence can be plotted (Fig. 5). In order to find analytical expression for engineering stress-engineering strain dependence we used Ogden model, which is a very general hyperelasticity model often used for description of non-linear stress-strain behavior of rubbers, polymers and other complex materials such as biological tissues\(^9\). The fitting function stemmed from an expression of uniaxial stress as function of stretch \( \lambda_i \), which is related to the strain according to \( \lambda_i = 1 + \varepsilon \).

\[
\sigma(\varepsilon) = \sum_{i=1}^{N} \frac{2 \mu_i}{a_i} \left[ (1+\varepsilon)^{a_i} - (1+\varepsilon)^{-0.5a_i} \right],
\]

where \( \mu \) and \( a \) represent material parameters. Figure 5 shows the fitting curve derived from Eq. (5) for \( N = 2 \). The parameters \( \mu_i \) and \( a_i \) were determined using Levenberg–Marquardt method\(^5\). Since the PMMA sample is uniaxially compressed along “y”-axis and there is no external stress applied along “x”-axis, i.e. free boundary condition applies, the difference in principal stresses \( \Delta \sigma \) in the “z”-plane equals the applied engineering stress \( \sigma \). Putting the \( \sigma(\varepsilon) \) dependence from Eq. (5) into Eqs. (3), (1) and (2), in this order, we get the intensity of transmitted light as function of the engineering strain \( \varepsilon \). This function was then used for fitting the measured dependences of intensity of light on the strain. It showed up that the parameters \( \mu_i \) and \( a_i \) determined by previous fitting had to undergo the fitting process again in order to get a good match between measured and calculated dependences. However, the values of parameters changed only slightly.
slightly. The measured and calculated dependences of intensity of light on strain for selected wavelengths are shown in Fig. 6.

![Figure 6. Measured and calculated normalized spectral distribution of light intensity for selected wavelengths in VIS (a) and NIR (b) spectral ranges as function of strain.](image)

It is obvious that from character of the dependences plotted in Fig. 6 one can conclude the same as we did in case of the dependences of intensity of light on the load, i.e. visible light is better to use only for sensing the lower loads and light from NIR spectral range can preferably be used for sensing higher loads.

5. CONCLUSION

We investigated the response of polymethylmethacrylate samples to applied load utilizing its birefringence. The samples were placed between two crossed plane polarizers and deformed by the machine for fatigue testing of materials. The investigation was performed in VIS and NIR spectral ranges and for loads from 0 to 20kN. The dependences of intensity of light transmitted through the sample on the load as well as the strain were obtained. The characters of analytical functions expressing the dependence of the PMMA birefringence on the load (force) and strain were suggested and used for calculation of the intensity of transmitted light. A good match between calculated and measured data was achieved and demonstrated for selected wavelengths in VIS and NIR spectral ranges. The results imply that obtained functions may be used as theoretical calibration curves for PMMA sensor element intended for sensing or detecting force, weight, normal pressure or deformation. Also, it follows from the results that sensor based on visible light would be more appropriate for detecting low loads and light from NIR spectral range would be preferably used for sensing higher loads.

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