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ABSTRACT

A 2D hot spot diffusion model for describing the superconducting nanowire response to single photon is presented. We develop the 1D hot belt model to 2D hot spot model and can capture the initial stage of the hot spot evolution after photon absorption, this is helpful to comprehensive understand the origins and underlying physics of time jitter in superconducting nanowire-based single-photon detectors. Furthermore, the developed model can qualitatively explain the asymmetry and photon wavelength dependent probability density function (PDF) of the delay time. We find that the left and right half of the PDF distributions respective exhibit the Gauss and non-Gauss shape as increasing the excitation wavelength or decreasing the bias current, and the origins are discussed and analyzed in the framework of newly developed 2D model. The proposed 2D hot spot diffusion model will not only sheds light on the origin and influence of the timing jitter but will also reveal the work principle of the superconducting nanowire devices.

Keywords: SNSPDs, time jitter, time delay, single photon detector, electrothermal model, 2D hot spot model, wavelength, superconducting detectors

1. INTRODUCTION

The excellent combined performances of superconducting nanowire-based single-photon detectors (SNSPDs), such as negligible dark counts, high detection efficiency, ultralow time jitter (timing accuracy) and ultrahigh count rates has made these devices very promising for various application fields. In particular, their unrivalled counting rates and time jitter ability has made them very suitable for time related applications, such as the quantum key distributions (QKD), satellite laser ranging, astronomy observations and life science. Some experimental studies as well as a few theoretical ones on the minimal time jitter has recently appeared. However, the origins and mechanism of the timing jitter is elusive and obstruct further improvement for more time demanding science and engineering applications.

Most of the theoretical detection models for determining the timing performance of SNSPDs mainly focus on the microscopic time dependent diffusion of the quasiparticles or the dynamics of the superconducting order parameter. Yang et. al6 firstly introduced a 1D electrothermal model to consider the electrical response with an equivalent electric circuit, and the coupled thermal properties was described by a 1D time-dependent heat diffusion differential equation. This phenomenology model was very successful in simulating the electrical response problem7 and some new device structures9 in SNSPD, but the oversimplification of the heat diffusion process with a single 1D differential equation lose the details.
of the microscopic current density redistribution and the hot spot evolution at the initial state of photon detection. In this paper, we expand the 1D hot belt model to 2D hot spot diffusion model, to describe the response properties of SNSPDs from both microscopic and phenomenology point of view. The cross-section effect induced from the different radial position the photons irradiated along the nanowire is taken into account, which can thereby capture the underlying physics of the recently photon wavelength dependent time jitter studies\textsuperscript{8-11}, and can qualitatively explain the asymmetry and non-Gaussian shape in the probability density function (PDF) distributions we observed recently.

Fig. 1. Schematic of the single photon detection mechanism of 100 nm width superconducting nanowire and $\sigma_w$ standard deviation.
The black arrows indicate the bias current density flow through the Au pad and the NbN nanowire, the red and gray symbols represent the generated hot spot and the defects on the nanowire. The temperature of the normal resistive region can be expressed by the marked formula.

2. NUMERICAL MODEL

The 2D hot spot diffusion model is the extension of the 1D hot belt model, and some new algorithms are introduced to solve the 2D partial differential equation. The basis of the 2D hot spot model is still the time-dependent heat conduction equation, but the equation adds a dimension in space as shown in Figure 1. Since the two ends of the nanowire are connected to the electrodes, the Norman condition of the temperature is taken for the left and right boundaries of the nanowire. Because the nanowires is in a vacuum atmosphere, under the premise of ignoring the thermal radiation, there is no heat exchange between the upper and lower boundaries of the nanowire and the outside atmosphere, so the temperature is taken as the Dirichlet boundary condition. The length of the nanowire is $L$ and the width is $w$. Then the left vertex of the nanowire is set as the origin, and the differential equation can be obtained as follows

$$\frac{\partial cT}{\partial t} = \frac{I^2}{\rho_0} + K \nabla \cdot \nabla T + \frac{\alpha}{\sigma}(T - T_p)$$

(1)

The boundary conditions can be expressed as:

$$T|_{x=0} = T_l$$

(2)

$$T|_{x=L} = T_r$$

(3)

$$\frac{\partial T}{\partial y}\bigg|_{y=0} = 0$$

(4)

$$\frac{\partial T}{\partial y}\bigg|_{y=W} = 0$$

(5)
The parameter $c$, $\rho_0$, $K$, $\alpha$ are state dependent, and the expressions for superconducting and normal state can refer to Yang et al. paper. Then we use the local one-dimensional scheme (LOD) to solve the Equation (1). The detail of the algorithms can see the Appendix A.

3. EXPERIMENTAL METHODS

The fabrication of the SNSPD devices starts with the magnetron sputtering of 6 nm thick NbN ultrathin film on the Si and Silicon oxide substrate. Then the 100 nm wide, 50% filling factor, 11x11 $\mu$m$^2$ active area meander nanowire is fabricated by a series of micro-nano processing, such as the optical lithography (OL), electron beam lithography (EBL), reactive ion etching (RIE), photoresist removing and dicing. The femtosecond laser is coupled to the nanowire detector through a HP780 single mode fibre. The packaged nanowire detector is installed on the second stage of a GM cryocooler (RDK-101D, Sumitomo), where the lowest temperature is 2.2 K, and can be changed by the temperature control unit (Lakeshore 325). The DC, RF and DC&RF port of a bias-tee (ZFBT-4R2GW+) are fed through the signal to the constant current supply, the readout circuit (TCSPC, Hydraparp400) and the nanowire detector, respectively. Two inverters (A-PPI-D, Becker & Hickl) and two 10 dB electric attenuators are added to transfer the electrical signal to the transistor-transistor logic (TTL) and the optimal input voltage range of about -200 mV of the TCSPC. The femtosecond laser coupling to the SNSPDs is based on a pumping-probe optical parametric amplifier (OPA) system, which can produce 240 nm to 15 $\mu$m wavelength photons with <120 fs pulse width and 5 kHz repetition frequency. The original laser is intensely attenuated by a premium bandpass optical filter (FWHM 4 nm) and four optical adjustable attenuators (0–340 times) to ensure the single photon detection regime for all wavelength photons. The picosecond laser with different pulse widths at different wavelengths is based on a commercial SuperK EXTREME supercontinuum laser (640-2400 nm tuneable wavelengths, 80 MHz repetition frequency) and a matching SuperK SELECT multi-line tunable filter.

4. RESULTS AND DISCUSSION

Figure 2a shows the experimental normalized histograms of the IRF (instrumental response function) with the excitation wavelengths of 532, 750, 980 and 1064 nm in the logarithmic scale. The data are measured at the bias current $0.9I_c$ and the base temperature 2.2 K. To facilitate comparison, we set the maximum delay time to zero for all four excitation wavelengths. The black arrow marks the shift direction of IRF distributions. The dashed line indicated the FWHM of the distributions, which is 56, 59, 63 and 64 ps for 532, 750, 980 and 1064 nm, respectively. It is obvious that the Gauss curve fits very well with the results of the 532 nm wavelength, both at the shorter (left half) or longer (right half) time delay than the maximum probability time delay (MPTD). However, as increasing the excitation wavelength, the right half are more and more deviate from the Gauss shape, with the left half remaining the same. The origins of the slightly ascending points for 532 nm of our results is unknown yet. Considering the much bigger effective area of our nanowire detectors than other groups, it maybe stem from the larger bends ratio of our meander nanowire, where the current crowding effect makes the bends of the meander nanowires response to the incident photons faster than the straight parts.

The increasing asymmetry and non-Gaussian shape of the PDF distributions for longer excitation wavelength and lower bias current were observed by some other groups recently, but the explanations were not reach to an agreement yet. Sidorova et al. attributed them to the different detection regimes for 800 and 1560 nm wavelengths. In our recent studies, we observed the longer delay time and wider FWHM distributions as decreasing the bias current, and the sudden deviation
from the Gauss shape at right half distributions for bias current <0.7Ic. The redshift of the MPTD and the broadening of the histogram distributions can explain in semi-quantitative in the framework of 1D hot belt model. Phenomenologically, the dimension of the initial normal region is proportional to the energy of the irradiated photons, so the 1D hot belt model ignored the cross-section effect. The 2D hot spot diffusion model we introduced can investigate the hot spot evolution process at the initial stage of the photon response and the voltage pulse generation, which might help to better understand the origins of the asymmetry of the PDF distributions.

![Graph](https://example.com/graph.png)

**Fig. 2.** (a) Experimental results of the PDF distributions at different excitation wavelengths in the logarithmic scale at 0.9Ic. The dashed line indicates the FWHM of the PDF distributions of the response delay time. (b) Calculated time delay of single photon detection as a function of radial position along the nanowire, with the parameters of 100 nm nanowire width, 5 nm nanowire thickness, 2.2 K operating temperature and changeable excitation wavelengths from 532 to 1064 nm.

Figure 2b and Figure 3 show the voltage pulse time delays after the arrival of the specific wavelength photons, versus to the different radial positions along the cross-section of nanowire. The computer modeled parameters of nanowire width, nanowire thickness and operating temperature are 100 nm, 5 nm and 2.2 K, respectively. The last column of Figure 3 represents the delay time required for the hot spot cutting off the whole sidewalk after the photons irradiated at the specific radial position along the nanowire. It is obviously that the delay time nonlinearly increase as the absorption site of the photons move from the center to the edge of the nanowire, and the increasing slope is inversely proportional to the excitation photon wavelengths. At the center position of the nanowire, the delay times of the photon response are nearly indistinguishable from 5.65 to 7.7 ps for 532 and 1064 nm, respectively. However, with the photons deviating 30 nm distance from the center position of the nanowire, the delay time is over 15 ps faster for 532 nm than 1064 nm excitation wavelength and is clearly separated for the median wavelengths of 750 and 980 nm. The initial hot spot temperature and diameter are chosen as 25 K and 32 nm for wavelength 532 nm according to the literature. For simplifying the simulation, we fixed the hot spot temperature on 25 K for the other three photon wavelengths and change the corresponding hot spot diameters proportional to their photon energies. This is reasonable considering the equivalence of the higher hot spot temperature and larger hot spot diameter for shorter excitation wavelength.

Introducing the uncertainty of the absorption position along the radial direction, the trends of the calculated time delays can be in good consistent with the experimental results. The photons irradiated at the center position corresponds to the coincide of the left half of the PDF distributions for each excitation wavelengths, and the proportional deviation from the
Gauss fit curve with increasing the wavelengths can be explained by the different delay time for each wavelength photons as they illuminate at the off-center positions along the nanowire. As illustrated in Figure 2b, the time delay variance between the photons irradiated at the center and 30 nm off the center are 21.95, 23.45, 28.35, 34.95 ps for 532, 750, 980 and 1064 nm excitation wavelength photons, respectively. Note that for wavelengths 532 and 750 nm, the effective radial position can reach the 35 and 34 nm off the center, respectively, which are inaccessible for the wavelengths 980 and 1064 nm. Because the hot spot model is only a phenomenology model, so it makes sense to ignore these points when we calculate the time jitter. For the sake of clarity, we do not mark the point of 532 nm photons irradiating in just the edges of the nanowire.

![Figure 3](https://example.com/fig3.png)

Fig. 3. The time evolution of the hot spot with the 532 nm photons irradiating at the different radial position along the nanowire. The third column represents the delay time required for the hot spot cutting off the whole sidewalk of the nanowire.

To further demonstrate the above explanations of the wavelength dependent time jitter, we use a supercontinuum white light laser with selected ultra-narrow wavelength to obtain the IRF curve and further investigate the origins of the asymmetry and non-Gaussian shape of the PDF distributions. The PDF distributions for the wavelengths 640, 750, 980 and 1064 nm as a function of the relative bias current are shown in Figure 4a. As in our recently paper, the MPTD and the time delay variance broadening (FWHM) both increase with the decreasing of the relative bias current, which can be explained as the longer time required for the hot spot region to spread cutting off the whole cross section at lower bias current. The asymmetry of the PDF distribution is also obvious in Figure 4b but something different with the results of the pump-probe femtosecond laser system. For the left half, the distributions of the PDF always obey the Gaussian pattern. The entirety shift of the distribution is reasonable because of the wider pulse width of the femtosecond laser for shorter
wavelengths. The non-Gauss pattern of the right half of the PDF distributions is also observed by some other groups, but the origins have not been elucidated. The slightly redshift for longer wavelength can explained using our developed 2D diffusion model as the longer time delay for the longer excitation wavelength with smaller hot spot diameter, and the quasi-exponential distribution for all wavelengths might be due to the Fano fluctuations of the deposited photon energies. The intrinsic detection efficiency (IDE) for the four wavelengths are shown in Figure 4c, the linear relation of the IDE with the relatively bias currents ensures the single photon detection region for all wavelengths.

![Figure 4](https://ebooks.spiedigitallibrary.org/conference-proceedings-of-spie/)

**Fig. 4.** (a) The PDF distributions for the wavelengths 640, 750, 980 and 1064nm as a function of the relative bias currents. (b) The normalized comparison of the PDF distributions for these four wavelengths at 0.9Ic. (c) The intrinsic detection efficiency (IDE) for these four wavelengths as a function of relative bias currents.

5. CONCLUSION

In summary, we systematically investigate the impact of cross-section effect on the timing performance of SNSPDs by introducing a newly 2D hot spot diffusion model. The simulations can qualitatively explain the time delay variance with changeable bias current and excitation wavelength observed recently by us and some other groups. The more asymmetry and deviate from the Gauss shape for longer wavelength photons is the result of the much faster increasing of the delay time as the photons irradiate far from the center position along the nanowire. We also find the different mechanisms responsible for the left and right half of the PDF distributions which need further investigations. The simulations of the timing performance of SNSPDs by our models will help to elucidate the mechanism of time jitter and provide some inspiration to further improve it for time demanding applications.
Appendix A: 2D hot spot diffusion model solution

The difference between the 2D model and the 1D model is not only in the increase in dimension but also in the introduction of new variables. The complexity of the electrothermal model lies in the state of the nanowires as a function of time and location and is determined by both temperature and current density. In the 1D model, since the width and depth of the nanowires are ignored, the current density at each point on the same cross section can be considered to be the same, so the current density can be directly replaced by the current flowing through the nanowire. This replacement is not only reasonable, but also eliminates solving the current density and greatly simplifies the solution process. The current itself is provided by the circuit equation, so this simplification does not increase the complexity of the solution in other aspects. However, for the 2D model, because the width of the nanowire is not negligible, the nanowire has a cross-sectional area. For each point on the same cross-section, the current density is different. In this case, instead of replacing the current density with the current flowing through the nanowire as in the 1D model, the current density must be solved independently to introduce a new variable—current density. It is worth noting that, compared to other variables, the current density is the most complicated to solve. This is why the complexity of the 2D model is much higher than that of the 1D model.

We use the LOD approach to substitute the Crank–Nicolson finite difference method in the 1D electrothermal method. The partial differential equation can be expressed as:

\[
\frac{1}{2} \frac{\partial T}{\partial t} = k \frac{\partial^2 T}{\partial x^2} - \frac{\alpha}{d} \frac{\partial T}{\partial t} + \frac{1}{2} \left( \frac{\alpha}{d} T_b + J^2 \rho \right) \tag{A1}
\]

\[
\frac{1}{2} \frac{\partial T}{\partial t} = k \frac{\partial^2 T}{\partial y^2} - \frac{\alpha}{d} \frac{\partial T}{\partial t} + \frac{1}{2} \left( \frac{\alpha}{d} T_b + J^2 \rho \right) \tag{A2}
\]

Which can then be derived to

\[
-\frac{K T}{2 \Delta x^2} T_{i-1,j}^{n+1/2} + \left( c + \frac{K T}{\Delta x^2} \right) T_{i,j}^{n+1/2} - \frac{K T}{2 \Delta x^2} T_{i+1,j}^{n+1/2} = \frac{K T}{2 \Delta x^2} T_{i-1,j}^n + \left( c - \frac{K T}{\Delta x^2} \right) T_{i,j}^n + \frac{K T}{2 \Delta x^2} T_{i+1,j}^n + \frac{\tau}{2} \left( J^2 \rho + \frac{\alpha}{d} T_b \right) \tag{A3}
\]

\[
-\frac{K T}{2 \Delta x^2} T_{i,j}^{n+1/2} + \left( c + \frac{K T}{\Delta x^2} \right) T_{i,j+1}^{n+1/2} - \frac{K T}{2 \Delta x^2} T_{i,j+1}^{n+1/2} = \frac{K T}{2 \Delta x^2} T_{i,j-1}^n + \left( c - \frac{K T}{\Delta x^2} \right) T_{i,j}^n + \frac{K T}{2 \Delta x^2} T_{i,j+1}^n + \frac{\tau}{2} \left( J^2 \rho + \frac{\alpha}{d} T_b \right) \tag{A4}
\]

To keep the convergence of the effective temperature in and out of the hot spot, we choose the step size of the time as 0.5 ps, and the corresponding current density can be expressed as I/ d (w-dim.).

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