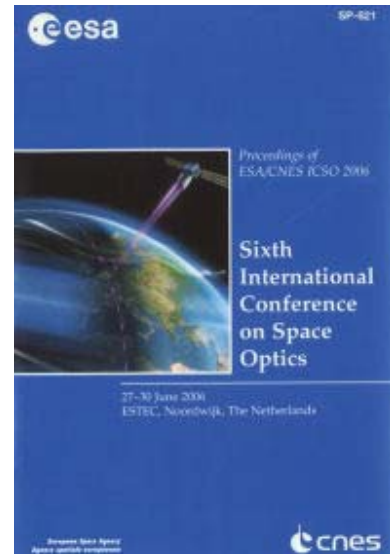


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Two mode optical fiber in space optics communication

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TWO MODE OPTICAL FIBER IN SPACE OPTICS COMMUNICATION

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ABSTRACT

In our contribution we propose to use of a two-mode optical fiber as a primary source in a transmitting optical head instead of the laser diode. The distribution of the optical intensity and the complex degree of the coherence on the output aperture of the lens that is irradiated by a step-index weakly guiding optical fiber is investigated. In our treatment we take into account weakly guided modes with polarization corrections to the propagation constant and unified theory of second order coherence and polarization of electromagnetic beams.

1. MOTIVATION

There are several basic methods that can be used for free space optic links (FSO) improvement with aspect of link availability. One of them is the use of partially coherent optical beams. This method is effective tools for suppressed of turbulence effects in atmosphere. This fact has been shown in many papers, for example in [7]. In our paper we would like demonstrate that optical fiber enables to generate a suitable partially coherent beam. Gauss-Schell beams (GSB) are ordinary considered in mentioned publications. But the fiber's beam and GSB beam are distinct from one to another in the shape and also in the coherence properties.

There is another reason for using an optical fiber for a generating of optical beams in FSO except the more immunity of fiber's beam towards the turbulence. The concept with the optical fiber as a primary source instead of the laser diode is compatible with pure photonic transmitting head, i.e. the head without electronic blocks. But our main goal of this paper is to describe the fiber's beam and to demonstrate that this beam has suitable properties towards to propagation through turbulent atmosphere.

2. MODE STRUCTURE

We start our analysis from the distribution of the electric field in the step-index optical fiber. We assume that this fiber is weakly guiding and that the fiber is situated in a Cartesian coordinate system so that the axis of the fiber coincides with z axis of the coordinate

system. In this case the electric field of the individual modes that are propagated in positive direction of the z -axis is described according these equations:

- Modes with $l = 0$:

$$\begin{aligned} \mathbf{E}_{0,m}^{(1)}(r, \varphi, z) &= \mathbf{e}_{0,m}^{(1)} \exp(-i\beta_{0,m}^{(1)}z) = \mathbf{x}_0 F_0(r) \exp(i\beta_{0,m}^{(1)}z), \\ \mathbf{E}_{0,m}^{(2)}(r, \varphi, z) &= \mathbf{e}_{0,m}^{(2)} \exp(-i\beta_{0,m}^{(2)}z) = \mathbf{y}_0 F_0(r) \exp(i\beta_{0,m}^{(2)}z), \end{aligned} \quad (1a)$$

- Other modes with $l > 0$:

$$\begin{aligned} \mathbf{E}_{l,m}^{(1)}(r, \varphi, z) &= \mathbf{e}_{l,m}^{(1)}(r, \varphi) \exp(i\beta_{l,m}^{(1)}z), \\ \mathbf{E}_{l,m}^{(2)}(r, \varphi, z) &= \mathbf{e}_{l,m}^{(2)}(r, \varphi) \exp(i\beta_{l,m}^{(2)}z), \\ \mathbf{E}_{l,m}^{(3)}(r, \varphi, z) &= \mathbf{e}_{l,m}^{(3)}(r, \varphi) \exp(i\beta_{l,m}^{(3)}z), \\ \mathbf{E}_{l,m}^{(4)}(r, \varphi, z) &= \mathbf{e}_{l,m}^{(4)}(r, \varphi) \exp(i\beta_{l,m}^{(4)}z), \end{aligned} \quad (1b)$$

$$\begin{aligned} \mathbf{e}_{l,m}^{(1)}(r, \varphi) &= F_l(r) [\cos(l\varphi)\mathbf{x}_0 - \sin(l\varphi)\mathbf{y}_0], \\ \mathbf{e}_{l,m}^{(2)}(r, \varphi) &= F_l(r) [\cos(l\varphi)\mathbf{x}_0 + \sin(l\varphi)\mathbf{y}_0], \\ \mathbf{e}_{l,m}^{(3)}(r, \varphi) &= F_l(r) [\sin(l\varphi)\mathbf{x}_0 + \cos(l\varphi)\mathbf{y}_0], \\ \mathbf{e}_{l,m}^{(4)}(r, \varphi) &= F_l(r) [\sin(l\varphi)\mathbf{x}_0 - \cos(l\varphi)\mathbf{y}_0], \end{aligned}$$

where

$$F_l(r) = \begin{cases} \frac{J_l\left(U_{l,m} \frac{r}{a}\right)}{J_l(U_{l,m})} & \text{for } r \leq a, \\ \frac{K_l\left(W_{l,m} \frac{r}{a}\right)}{K_l(W_{l,m})} & \text{for } r > a, \end{cases} \quad (2)$$

where J_l and K_l are Bessel function of the first kind and second kind of the order l respectively, a is a core radius, $U_{l,m}$ and $W_{l,m}$ are modal parameters for the core and cladding, $\beta_{l,m}^{(q)}$ is the propagation constant and r, φ, z are variables in the polar coordinate system and i is the imaginary unit. The symbol $\mathbf{E}_{l,m}^{(q)}$ stands for a complex representation of a real monochromatic electrical vector of the angular frequency ω , nevertheless the multiplicative factor $\exp(-i\omega t)$ in equations (1a) and (1b) was omitted.

Accordingly expressions (1), there exist the group of four modes for every modal numbers l and m if $l > 0$. We are distinguished these modes by a superscript (q). If $l = 0$ then there are only two modes for modal numbers l and m . The propagation constant $\beta_{l,m}^{(q)}$ and the group velocity $g_{l,m}^{(q)}$ of individual modes are possible to express as [1]

$$\beta_{l,m}^{(q)} = \beta_{l,m} + \delta\beta_{l,m}^{(q)}, \quad g_{l,m}^{(q)} = g_{l,m} + \delta g_{l,m}^{(q)}, \quad (3)$$

where $\beta_{l,m}$ is a scalar propagation constant (the solution of scalar characteristic equation), $\delta\beta_{l,m}^{(q)}$ is the polarization corrections, $g_{l,m}$ is the group velocity derived from purely scalar treatment and $\delta g_{l,m}^{(q)}$ is the polarization correction to the group velocity.

The scalar propagation constant is the same for modes that are differ only by the index q , but polarization corrections may be different in this case. Precisely, the modal parameters $U_{l,m}$ and $W_{l,m}$ are also different for every mode and they should be denote also by the superscript q . However, we didn't take into considerations the polarization corrections for modal parameters, because their influence is unimportant.

Tab. 1: Parameters of modes in SMF-28 fiber at operating wavelength 830 nm.

	$\mathbf{E}_{0,1}^{(1)}, \mathbf{E}_{0,1}^{(2)}$	$\mathbf{E}_{1,1}^{(1)}, \mathbf{E}_{1,1}^{(3)}$	$\mathbf{E}_{1,1}^{(2)}$	$\mathbf{E}_{1,1}^{(4)}$
$U_{l,m} [-]$	1,87	2,92		
$W_{l,m} [-]$	3,16	2,23		
$\beta_{l,m} [\text{m}^{-1}]$	11101500,1	11088305,5		
$g_{l,m} [\text{m} \cdot \text{s}^{-1}]$	204302892,3	204216699,3		
$\delta\beta_{l,m}^{(q)} [\text{m}^{-1}]$	-11,99	-28,6	-27,4	0,0
$\delta g_{l,m}^{(q)} [\text{m} \cdot \text{s}^{-1}]$	-50,9	-596,7	-792,4	-505,3

In our analysis we are considered the two mode fiber, concretely the fiber SMF-28 manufactured by a Corning and operating at the wavelength 830 nm. We computed all mentioned quantity for this fiber and we summarize them in the previous table, Tab.1. The modes $\mathbf{E}_{0,1}^{(1)}$ and $\mathbf{E}_{0,1}^{(2)}$ are different only in the polarization and all other parameters as propagation constant and group velocity are the same. Similarly, modes $\mathbf{E}_{1,1}^{(1)}$ and $\mathbf{E}_{1,1}^{(3)}$ have the same group velocity and propagation constant but are different in the spatial configuration, see Eq. (1). The numeration of modes by the help of subscripts l, m and superscript q is often

unfavourable in a mathematical formula, therefore we introduce a single subscript j for every mode, so the j -th mode is marked as \mathbf{E}_j . The assignment is obvious from the following table, Tab. 2.

Tab. 2: The assignment between usual indexing of modes in two mode fiber and indexing by the help of single subscript.

Mod	$\mathbf{E}_{0,1}^{(1)}$	$\mathbf{E}_{0,1}^{(2)}$	$\mathbf{E}_{1,1}^{(1)}$	$\mathbf{E}_{1,1}^{(2)}$	$\mathbf{E}_{1,1}^{(3)}$	$\mathbf{E}_{1,1}^{(4)}$
j	1	2	3	4	5	6

We are also indexing by the subscript j parameters that belong to the individual mode, for example propagation constant β_j , group velocity g_j , etc..

3. THE COHERENCE OF LIGHT IN AN OPTICAL FIBER

The basic function for description of second-order coherence phenomena in the scalar theory of random process is the mutual coherence function [2]. In our case we must consider not scalar field but a vector field according (1), hence we must use the theory that enables the analysis of such field. The pure scalar description of the coherence phenomena in the optical fiber is described in [3]. For description of an electromagnetic beam was introduced the cross-spectral density matrix $\underline{\mathbf{W}}$ defined in [4]. The cross-spectral density matrix describes the coherence phenomena in a space-frequency domain. The description in a space-time domain is possible by the means of the mutual coherence matrix $\underline{\mathbf{\Gamma}}(\mathbf{r}_1, \mathbf{r}_2, \tau)$, defined in [6], whose elements are

$$\underline{\mathbf{\Gamma}}(\mathbf{r}_1, \mathbf{r}_2, \tau) = \begin{bmatrix} \Gamma_{xx}(\mathbf{r}_1, \mathbf{r}_2, \tau) & \Gamma_{xy}(\mathbf{r}_1, \mathbf{r}_2, \tau) \\ \Gamma_{yx}(\mathbf{r}_1, \mathbf{r}_2, \tau) & \Gamma_{yy}(\mathbf{r}_1, \mathbf{r}_2, \tau) \end{bmatrix}, \quad (4)$$

$$\Gamma_{\alpha\beta}(\mathbf{r}_1, \mathbf{r}_2, \tau) = \langle E_{\alpha}^*(\mathbf{r}_1, t) E_{\beta}(\mathbf{r}_2, t + \tau) \rangle, \quad (\alpha, \beta = x, y),$$

where $\mathbf{E}(\mathbf{r}, t) = \mathbf{x}_0 E_x(\mathbf{r}, t) + \mathbf{y}_0 E_y(\mathbf{r}, t)$ stands for an ensemble of the complex representation of the real electric vector again, but now for a polychromatic light. The brackets $\langle \rangle$ denote the ensemble average, but because we are assumed that the electric field is stationary and ergodic, these brackets mean also the time average. The elements of these two matrices $\underline{\mathbf{W}}$ and $\underline{\mathbf{\Gamma}}$ are connected by the Fourier transform (Weiner-Khintniche theorem) [2]

$$\Gamma_{\alpha\beta}(\mathbf{r}_1, \mathbf{r}_2, \tau) = \int_0^{\infty} W_{\alpha\beta}(\mathbf{r}_1, \mathbf{r}_2, \tau) \exp(-i\omega\tau) d\omega, \quad (5)$$

$$W_{\alpha\beta}(\mathbf{r}_1, \mathbf{r}_2, \omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \Gamma_{\alpha\beta}(\mathbf{r}_1, \mathbf{r}_2, \tau) \exp(i\omega\tau) d\tau.$$

We can express the resultant electric field at the end of the optical fiber, whose length is z , as

$$\mathbf{E}(\mathbf{r}, z, t) = \sum_j \int_0^{\infty} a_j(\omega) \mathbf{e}_j(\mathbf{r}, \omega) \exp[i\beta_j(\omega)z - i\omega t] d\omega, \quad (6)$$

where the summation is meant for all guided modes, $a_j(\omega)$ is a modal weight of the j -th mode, \mathbf{r} is a position vector in the cross-section in the fibre (in a plane $z = const.$) and $\mathbf{e}_j(\mathbf{r}, \omega)$ represents the electric vector of j -th mode, see Eq. (1), where the dependence of angular frequency wasn't introduce. If we substitute the expression (6) into (4) then we obtain the mutual coherence matrix with elements

$$\Gamma_{\alpha\beta}(\mathbf{r}_1, \mathbf{r}_2, z, \tau) =$$

$$= \sum_j \sum_k \int_0^{\infty} \int_0^{\infty} \langle a_j^*(\omega) a_k(\omega') \rangle e_{j,\alpha}^*(\mathbf{r}_1, \omega) e_{k,\beta}(\mathbf{r}_2, \omega') \times \quad (7)$$

$$\times \exp[i\omega'(t + \tau) - i\omega t] \exp\{i[\beta_k(\omega') - \beta_j(\omega)]z\} d\omega d\omega',$$

where $e_{j,\alpha}$ means the α -component of the j -th mode. The modal weight $a_j(\omega)$ is possible to determine from the knowledge of the distribution of electrical field at the input of optical fibre [1]

$$a_k(\omega) = \frac{1}{N_k} \iint_S \mathbf{E}_s(\mathbf{r}, 0, \omega) \cdot \mathbf{e}_k^*(\mathbf{r}, \omega) d^2\mathbf{r}, \quad (8)$$

$$N_k = \iint_S \mathbf{e}_k(\mathbf{r}, \omega) \cdot \mathbf{e}_k^*(\mathbf{r}, \omega) d^2\mathbf{r}.$$

The symbol $\mathbf{E}_s(\mathbf{r}, 0, \omega)$ in this equation (8) signifies the electric vector at the input of the optical fiber and so this vector is given by the exciting source. We integrate in (8) over the infinite cross-section of the optical fiber. The equation (8) is valid for the weakly guiding fiber only. If we substitute the expression for a_j and a_k from (8) to the equation (7) we obtain

$$\Gamma_{\alpha\beta}(\mathbf{r}_1, \mathbf{r}_2, z, \tau) = \sum_j \sum_k \int_0^{\infty} \int_0^{\infty} e_{j,\alpha}^*(\mathbf{r}_1, \omega) G_{jk}(\omega) e_{k,\beta}(\mathbf{r}_2, \omega) \times \quad (9)$$

$$\times \exp(i\omega\tau) \exp\{i[\beta_k(\omega) - \beta_j(\omega)]z\} d\omega,$$

where

$$G_{jk}(\omega) = \frac{\iiint_S \mathbf{e}_j(\mathbf{r}_1, \omega) \underline{\mathbf{W}}_s(\mathbf{r}_1, \mathbf{r}_2, \omega) \mathbf{e}_k^H(\mathbf{r}_2, \omega) d^2\mathbf{r}_1 d^2\mathbf{r}_2}{N_j N_k}, \quad (10)$$

where $\underline{\mathbf{W}}_s(\mathbf{r}_1, \mathbf{r}_2, \omega)$ is the cross-spectral density matrix at the input of the fiber, superscript H means the transpose and conjugate matrix and we are introduced the matrix for the electric vector of j -th mode

$$\underline{\mathbf{e}}_j(\mathbf{r}_1, \omega) = \begin{bmatrix} e_{j,x}(\mathbf{r}_1, \omega) & e_{j,y}(\mathbf{r}_1, \omega) \end{bmatrix}. \quad (11)$$

During the derivation of the (9) we assumed that the fiber is excited by the stationary and ergodic source.

3.1 The excitation of the fiber by the laser diode

The equation (9) is the desired expression for the mutual coherence matrix at the end of the optical fiber of the length z . We simplify this result provided the exciting source is a cross-spectrally pure quasi-monochromatic and spatial coherent. In this case, the cross spectral density matrix of the source is given by

$$\underline{\mathbf{W}}_s(\mathbf{r}_1, \mathbf{r}_2, \omega) = \underline{\mathbf{J}}_s(\mathbf{r}_1, \mathbf{r}_2) S(\omega), \quad (12)$$

$$J_{\alpha\beta}(\mathbf{r}_1, \mathbf{r}_2) = E_{\alpha}^*(\mathbf{r}_1) E_{\beta}(\mathbf{r}_2), \quad (\alpha, \beta = x, y),$$

where $S(\omega)$ is the spectral density of the source and $\underline{\mathbf{J}}_s(\mathbf{r}_1, \mathbf{r}_2)$ is the mutual intensity (or equal-time mutual coherence matrix) of the source. This case complies with the excitation of the optical fiber by the laser diode. The spectral density $S(\omega)$ of the quasi-monochromatic source is negligible if the angular frequency isn't near by a center frequency ω_0 . For that reason we can integrate in (9) only in a vicinity of the center frequency ω_0 . Further, for that the same reason, we ignore the frequency dependence of the electric vector of the individual modes, hence we are putting

$$\mathbf{e}_j(\mathbf{r}, \omega) = \mathbf{e}_j(\mathbf{r}, \omega_0) \equiv \mathbf{e}_j(\mathbf{r}) \quad \text{for all } j. \quad (13)$$

The frequency dependence of the propagation constant cannot be ignored, so we expand the propagation constant in a Taylor series and then we are considered only first two terms

$$\beta_j(\omega) = \beta_j(\omega_0) + (\omega - \omega_0) g_j^{-1}(\omega_0) = \beta_j + (\omega - \omega_0) \tau_j, \quad (14)$$

where $\beta_j(\omega_0) \equiv \beta_j$, $\tau_j(\omega_0) \equiv \tau_j$ is the group delay of the j -th mode for a fiber of the length 1 meter. Further, we take into account the natural shape of the spectral

line of the laser diode, hence the spectral density of the source is

$$S(\omega) = \frac{\Delta\omega}{\pi[\Delta\omega^2 + (\omega - \omega_0)^2]}, \quad (15)$$

where the $\Delta\omega$ is the width of the spectral line in the angular frequency domain. If we substitute from (12) and (14), (15) into (9) we obtain

$$\Gamma_{\alpha\beta}(\mathbf{r}_1, \mathbf{r}_2, z, \tau) = \sum_j \sum_k a_j^* a_k e_{j,\alpha}^*(\mathbf{r}_1) e_{k,\beta}(\mathbf{r}_2) \exp[i(\beta_k - \beta_j)z] \exp(-i\omega_0\tau) \gamma_s(\tau + \Delta\tau_{jk}z), \quad (16)$$

where $a_j = a_j(\omega_0)$ for all j , $\Delta\tau_{jk} = \tau_j - \tau_k$ and γ_s stands for the complex degree of temporal coherence of the source

$$\gamma_s(\tau) = \int_0^\infty S(\omega) \exp(-i\omega\tau) d\omega = \exp(-|\tau| \tau_s^{-1} - i\omega_0\tau), \quad (17)$$

where τ_s is the coherence time of the exciting source, that is approximately equal to the reciprocal value of the spectral line bandwidth in the angular frequency domain $\Delta\omega$, [2]. We can easily compute the cross-spectral density matrix $\underline{\mathbf{W}}$ from the knowledge of the mutual coherence matrix (16) by the help of relations (5), the result of this computation is

$$W_{\alpha\beta}(\mathbf{r}_1, \mathbf{r}_2, z, \omega) = \sum_j \sum_k a_j^* a_k e_{j,\alpha}^*(\mathbf{r}_1) e_{k,\beta}(\mathbf{r}_2) S(\omega) \times \exp[i(\omega_0 - \omega)\Delta\tau_{jk}z] \exp[i(\beta_k - \beta_j)z]. \quad (18)$$

4. PROPAGATION OF THE CROSS-SPECTRAL DENSITY MATRIX

Now, we determine the cross spectral density matrix on the output aperture of the lens that is irradiated by the optical fiber. The end of the fiber is placed in the plane $z = 0$. The fiber length will be marked by the symbol d . The end of the fiber coincides with the focal plane of the lens, concurrently the focal length of the lens is f . The propagation of the cross-spectral density matrix through the paraxial optical system obeys the propagation law

$$W_{\alpha\beta}(\mathbf{p}_1, \mathbf{p}_2, z, \omega) = \frac{\exp[D(\mathbf{p}_1^2 - \mathbf{p}_2^2)]}{(\lambda B)^2} \iiint_{S S} W_{\alpha\beta}(\mathbf{r}_1, \mathbf{r}_2, d, \omega) \times \exp\left\{\frac{ik}{2B} [A(\mathbf{r}_2^2 - \mathbf{r}_1^2) + 2(\mathbf{r}_1 \cdot \mathbf{p}_1 - \mathbf{r}_1 \cdot \mathbf{p}_2)]\right\} d^2\mathbf{r}_1 d^2\mathbf{r}_2, \quad (19)$$

where A, B, C and D are elements of the transfer matrix.

We can integrate separately in (19), i.e. at first according to variables r_1, φ_1 and then according to r_2 and φ_2 . Because the cross-spectral density of the matrix is expressed as the definite sum we can integrate term by term. If we use the relation (5) we obtain after some simplifications the mutual intensity matrix

$$\Gamma_{\alpha\beta}(\rho_1, \phi_1, \rho_2, \phi_2, z, \tau) = \sum_j \sum_k \Gamma_{jk, \alpha\beta}(\rho_1, \phi_1, \rho_2, \phi_2, z, \tau), \quad (20)$$

$$\Gamma_{jk, \alpha\beta}(\rho_1, \phi_1, \rho_2, \phi_2, z, \tau) = I_{j,\alpha}^+(\rho_1, \phi_1) I_{k,\beta}^-(\rho_2, \phi_2) \times \frac{a_j^* a_k}{(\lambda B)^2} \exp[i(\beta_k - \beta_j)d] \exp(-i\omega_0\tau) \gamma_s(\tau + \Delta\tau_{jk}d),$$

where

$$I_{j,\alpha}^\pm(\rho, \phi) = \int_{r=0}^\infty \int_{\varphi=0}^{2\pi} r e_{j,\alpha}^*(r, \varphi) \times \exp\left\{\pm \frac{ikA}{B} \left[\frac{r^2}{2} - r \rho \cos(\varphi - \phi)\right]\right\} d\varphi dr, \quad (21)$$

Integrals in (20) are the same except the sign before the imaginary unit, hence we distinguished them by the relevant superscript. From the relation (20) we can compute the most interesting quantity, as the optical intensity, complex degree of mutual intensity, degree of polarization, etc. We will be concentrated above all on the optical intensity and on the complex degree of spatial coherence.

4.1 The distribution of the optical intensity

If we are known the mutual coherence matrix we can calculate the optical intensity according to

$$I(\mathbf{p}, z) = \text{Tr}\{\underline{\mathbf{I}}(\mathbf{p}, \mathbf{p}, z, 0)\}, \quad (22)$$

where the Tr means the trace of the matrix.

The rotary symmetry of the optical intensity, i.e. the independence of the optical intensity on the angular variable ϕ , is one of the natural requirements. It is evidently, that the optical intensity has the rotary symmetry at the output aperture of the lens provided the optical intensity has rotary symmetry also at the end of the optical fiber. The optical intensity at the end of the optical fiber can be expressed on the basis of (16) and (22), after some algebraic modifications, as

$$I(\mathbf{r}, d) = \sum_j I_j(\mathbf{r}) + \sum_{\substack{j,k \\ j \neq k}} A_j A_k \mathbf{e}_j(\mathbf{r}) \cdot \mathbf{e}_k(\mathbf{r}) \times \left| \gamma_s(d \Delta\tau_{jk}) \right| \cos[\alpha_j - \alpha_k + (\beta_j - \beta_k)d], \quad (23)$$

where d is the length of the fiber, A_j and α_j are the modulus and the phase of the modal weight a_j , I_j is the optical intensity of the individual modes and $\Delta\tau_{jk} = \tau_j - \tau_k$.

Further, for simplicity we will be considered only guided modes with subscript $j = 1, 3, 4$. We can easily reach this situation if we are exciting the fiber by the slightly tilted linearly polarized beam. Details about this excitation of the optical fiber are introduced in [1] page 431. All of the propagating modes $j = 1, 3, 4$ have a different group velocity, see tab. 1, but because the polarization corrections δg_j are very small, i.e. it's valid $\delta g_j \ll g_j$, we can divide the optical fiber into three regions accordingly to the value of the argument $d\Delta\tau_{jk}$.

Region I: The condition (24) is valid in this case

$$|d\Delta\tau_{jk}| \gg \tau_s, \forall j, k, j \neq k. \quad (24)$$

Then the optical intensity is approximately

$$I(\mathbf{r}, d) = I(\mathbf{r}) = \sum_{j=1,3,4} I_j(\mathbf{r}). \quad (25)$$

The conditions (24) is valid for all supposed modes when

$$d \gg d_I = \frac{\tau_s}{\min|\Delta\tau_{jk}|} = \frac{\tau_s}{|\Delta\tau_{34}|} \cong 63 \text{ m}. \quad (26)$$

The calculations in (26) was executed for the parameters of the mode given by the Tab. 1 and for the laser diode source with 1 nm spectral line width.

Region II: For this case is valid the condition

$$|d\Delta\tau_{13}| \cong |d\Delta\tau_{14}| \gg \tau_s, \quad (27)$$

but we haven't any restriction between group delays of the modes \mathbf{e}_3 and \mathbf{e}_4 . Due to (17) hold true

$$|\gamma_s(d\Delta\tau_{13})| \cong |\gamma_s(d\Delta\tau_{14})| \cong 0. \quad (28)$$

The conditions (27) is valid if

$$d \gg d_{II} = \frac{\tau_s}{|\Delta\tau_{13}|} \cong \frac{\tau_s}{|\Delta\tau_{14}|} \cong 0,19 \text{ m}, \quad (29)$$

The optical intensity at the end of the optical fiber is then given by

$$I(\mathbf{r}, d) = \left\{ \sum_{j=1,3,4} I_j(\mathbf{r}) \right\} + 2A_3A_4 \mathbf{e}_3(\mathbf{r}) \cdot \mathbf{e}_4(\mathbf{r}) \times \\ \times |\gamma_s(d\Delta\tau_{34})| \cos[\alpha_3 - \alpha_4 + (\beta_3 - \beta_4)d] \quad (30)$$

There is the interferential term in (30) but this term can be disappeared if the value of the cosine term in (30) is zero, i.e. if the length of the fiber fulfills

$$d_n = \left| \frac{(n+1)\frac{\pi}{2} - \alpha_3 + \alpha_4}{(\beta_3 - \beta_4)} \right|, n \text{ is integer and } n \neq -1. \quad (31)$$

For discrete length of the fiber d_n according to (31) the optical intensity is given by the relation (25) and it's the exactly same as for the Region I.

Region III: For this case we cannot do any simplification of the relation (23) on the basis of the value of the argument $d\Delta\tau_{jk}$. In the Region III is impossible to obtain the rotary symmetrical distribution of the optical intensity.

Generally, we can say that in the Region I the guided modes don't interfere, in the Region II interfere only the modes with the same mode numbers l, m and finally all guided mode interfere in the Region III.

5. GRAPHICAL ILLUSTRATIONS

We demonstrate the distribution of the optical intensity and modulus of the complex degree of spatial coherence on the output aperture of the lens irradiated by the two mode optical fibre. The transfer matrix of the optical system is

$$\begin{pmatrix} A & B \\ C & D \end{pmatrix} = \begin{pmatrix} 1 & f \\ -\frac{1}{f} & 0 \end{pmatrix}. \quad (32)$$

The length of the optical fiber was chosen either in the Region I or in the Region II, but in this case with the discrete length equal to the length d_n according to (31). The optical intensity is determined by the expression (22), where $\underline{\Gamma}$ is given by (20). The complex degree of spatial coherence on the output aperture of the lens is defined

$$\gamma(\mathbf{p}_1, \mathbf{p}_2, f) = \frac{\text{Tr}\{\underline{\Gamma}(\mathbf{p}_1, \mathbf{p}_2, f, 0)\}}{\sqrt{\text{Tr}\{\underline{\Gamma}(\mathbf{p}_1, \mathbf{p}_1, f, 0)\}} \sqrt{\text{Tr}\{\underline{\Gamma}(\mathbf{p}_2, \mathbf{p}_2, f, 0)\}}}, \quad (33)$$

where elements of the $\underline{\Gamma}$ are given by (20) again.

The optical intensity is depicted on the Fig. 1. The modulus of the complex degree of spatial coherence is showed (Fig. 2) always for axial symmetric points $\rho_2 = -\rho_1$ on the figure Fig. 2. So it's the function only of the two variables ρ and ϕ

$$\rho = |\rho_1| = |\rho_2|, \quad \phi = \phi_1. \quad (34)$$

But in our choice of the length of the fiber the complex degree of spatial coherence is independent on the variable ϕ . The power carried by the mode $E_{0,1}^{(1)}$ is P_{01} and the power of the pair of modes $E_{1,1}^{(3)}$ and $E_{1,1}^{(4)}$ is P_{11} , concurrently the power of the these individual modes is the same. Integrals in (21) were calculated numerically by the help of the program MATLAB.

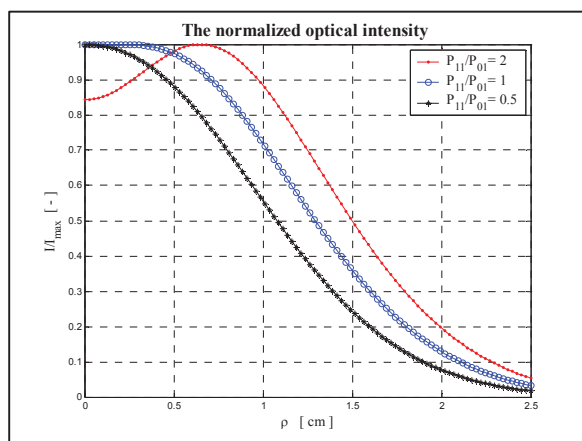


Fig. 1. The relation of the optical intensity on the radial distance ρ , the ratio of the power between individual modes is a parameter, $f = 20$ cm .

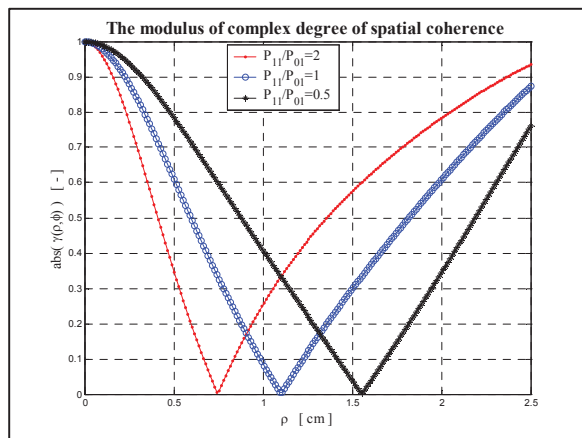


Fig. 2. The relation of the modulus of the complex degree of spatial coherence on the radial distance ρ , the ratio of the power between individual modes is a parameter, $f = 20$ cm .

6. CONCLUSIONS

We investigated coherence properties of the light on the output aperture of the lens that is irradiated by the weakly-guiding optical fibre. In our treatments we considered also polarization properties of the electric field. We divided the optical fibre on the basis of the comparison of the coherence time of the source and the time delay of the individual modes into three regions. The two-mode optical fibre is the suitable light source for FSO only in Region I and Region II, because only in these regions the optical intensity can be rotary symmetrical. It is apparent from Fig. 1 and Fig. 2 that the properties of the fibre's beam are noticeably different from Gauss-Schell model of the beam. There exist points, see Fig. 2, that are completely non-coherent, i.e. the modulus of the complex degree of the spatial coherence for these points is zero. The distribution of the optical intensity at the end of the optical fibre whose length belong to the Region I is independent on the propagation constants of the modes, hence the distributions of optical intensity is also independent on vibrations, temperature variations etc. in contrast to the Region II.

7. REFERENCES

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