# The method of impact point location based on planar double disk array

Wei Chen<sup>\*,a</sup>, Song Gao<sup>a</sup>, Guangming Fan<sup>a</sup>, Guangke Luo<sup>b</sup>, Haikun Zhang<sup>c</sup>, Yanglong Jin<sup>d</sup>, Xiaodong Lin<sup>e</sup>

 <sup>a</sup>Aerial Ammunition Support Department, Air Force Logistics Academy, Xuzhou China 221000;
 <sup>b</sup>Troops 94754, Jiaxing China 314000; <sup>c</sup> Nanjing Station, Nanjing China 210000; <sup>d</sup> Troops 95178, Nanning China 530000; <sup>e</sup> Shanghai Base, Shanghai China 200000.

#### ABSTRACT

How to accurately determine the location of the impact point is an urgent problem in many application fields. By analyzing the influence of oblique incidence of the projectile on the positioning accuracy, a planar double disk positioning array is proposed. The disk array is used to correct the error caused by oblique incidence of the projectile. The mathematical derivation and error analysis of the model are carried out. The correctness of the model algorithm and its positioning performance are verified through simulation experiments.

Keywords: Planar double disk array, sensor, impact point

## **1. INTRODUCTION**

Passive acoustic location technology is a technology that uses acoustic and electronic devices to receive sound field information to determine the position of target sound source. It belongs to passive location technology of radiation source. Its feature is that the system itself only receives the sound signal from the target source and uses the received signal to detect, track and locate the target<sup>1</sup>. This technology mainly uses the time difference (referred to as TDOA) between microphones or microphone arrays due to different signal propagation distances to achieve target location. Therefore, if the TDOA of acoustic waves reaching each array element can be accurately estimated, and according to the geometric relationship of microphone array layout, the parameter estimation of target location can be calculated. The planar double disk positioning array is one of the key technologies of passive acoustic positioning technology. The technical scheme of the planar double disk sensor array is to place several acoustic sensors on each disk. By establishing a mathematical model, each disk is equivalent to a vector sensor, which is used to determine the direction of the acoustic target. The intersection of the target directions measured by the two disks is the impact point<sup>2-3</sup>. In this paper, the mathematical derivation and error analysis of the planar double disk positioning array algorithm are carried out, and the simulation program is compiled with Matlab. Finally, the computer numerical simulation experiment is carried out to verify the performance of the positioning algorithm, which has good application value.

## 2. MEASUREMENT OBJECT OF PASSIVE ACOUSTIC POSITIONING AT IMPACT POINT

The projectile flying at high speed in the air, due to the energy transfer from the kinetic energy of the projectile to the air, generates an off body shock wave H at the head of the projectile. The front and back tracks of the shock wave form a cone with the apex at the head of the projectile as shown in Figure 1. The wave front is conical and moves at sound speed in the direction perpendicular to the wave front. The half angle of the cone  $\mu = \sin^{-1}(1/Ma_p)$  depends on the  $Ma_p$  Mach number of the supersonic projectile.

\*chenweijiangyiwa@126.com

Seventh International Conference on Mechatronics and Intelligent Robotics (ICMIR 2023), edited by Srikanta Patnaik, Tao Shen, Proc. of SPIE Vol. 12779, 1277920 · © 2023 SPIE · 0277-786X · Published under a Creative Commons Attribution CC-BY 3.0 License · doi: 10.1117/12.2688894 When the projectile shock wave sweeps through the detection point, its air pressure rapidly increases from static pressure  $P_0$  to overpressure  $P_0 + P_1$ , decays to secondary pressure  $P_0 - P_2$  with time and space, and finally recovers to  $P_0$ , forming a N wave signal as shown in Figure 2. Its complex disturbance may involve 3 or 4 pressure fluctuations.

The amplitude of the N wave and the width between the two amplitudes depend on the characteristics of the projectile, the velocity of the projectile and the normal distance from the trajectory to the detection point<sup>4-6</sup>.



Projectile / Tail compression wave T

Figure 1. Aerodynamic model of projectile



Figure 2. N waveform

When locating a projectile in flight, whether the projectile is supersonic or not, a specific sound wave will be generated. The acoustic signal can be preprocessed first, and then the TDOA can be extracted. For the time difference extraction of continuous wave signal, the commonly used method is to extract the time difference information by cross correlation operation. However, traditional cross correlation and cross power spectrum processing methods are very sensitive to noise, and their performance is greatly affected by noise and interference<sup>7-8</sup>.

## 3. MODEL AND ALGORITHM OF 3 PLANE DOUBLE DISK POSITIONING ARRAY

The double disk array uses two disk sensor arrays to achieve positioning. N acoustic sensor are evenly placed on the circumference of each disk (8 sensors are placed in Figure 3), and one acoustic sensor is placed in the center of each disk. When measuring the bullet flight, the shock wave generated or specific sound wave reaches the time difference of each sensor for positioning. In the automatic target reporting system of small arms, the impact point is located by measuring the shock wave generated during bullet flight<sup>9-10</sup>.

As shown in Figure 3, the center of the reference coordinate is the midpoint of the line between the centers of two disks, and the direction of the line between the centers of two disks is the X axis. 8 sound sensors  $S_1, S_2, \dots, S_{16}$  are placed at equal intervals on each disc, and one sound sensor  $S_{O1}$  and one sound sensor  $S_{O2}$  are respectively installed on the center  $O_1$  and  $O_2$ . Let the radius of the disc be r, the distance between the centers of two discs  $O_1O_2 = D$ .

$$x_{i} = r \cos((i-1) \times \theta) - D/2$$

$$y_{i} = r \sin((i-1) \times \theta)$$
(1)



Figure 3. Arrangement of acoustic sensor

Similarly, the calculation formula of coordinates of each sensor ( $S_9, \dots, S_{16}$ ) of disc 2 can be obtained, as shown in formula (2):

$$x_{i+8} = r\cos((i-1)\times\theta) + D/2$$
  

$$y_{i+8} = r\sin((i-1)\times\theta)$$
(2)

Among them,  $i = 1, \dots, 8$ ,  $\theta = \frac{360}{8} = 45^{\circ}$ .

The coordinates of sensor  $S_{o1}$  and  $S_{o2}$  are (-D/2, 0) and (D/2, 0) respectively.

The positioning algorithm of the planar double disk positioning array can be summarized as follows:

#### (1) Determine azimuth

When determining the azimuth, an approximation condition is used, that is, the approximation (taking the case of Figure 3 as an example)  $\angle O_1 BS_2$  and the right angle  $\angle O_2 DS_{12}$  (the simulation analysis will prove that this approximation error is very small later).

When the sound wave generated by the sound source P is transmitted to disc 1 and disc 2, the sensor will be triggered, and each sensor will record a moment  $t_1, t_2, \dots, t_{16}$ ,  $t_{SO1}$ ,  $t_{SO2}$ , for disc 1, there must be a minimum moment in the middle  $t_1, t_2, \dots, t_8$ , assuming that at a certain minimum moment  $T_{\min} = t_2$ , the connecting line from the sound source Pto the sensor  $S_{O1}$  intersects disc 1 at a point A, and the line segment PA is the shortest distance from the sound source P to disc 1, and the point is the nearest point on the circumference of the sound source P to disc 1, the point where the sensor is located is the point from the sound source P to the nearest sensor on disc 1. Generally,  $PS_2 \ge PA$ , if the triangle  $BO_1S_2$  is approximately regarded as a right triangle, the angle can be approximately expressed as:

$$\cos\theta_{1} = \frac{BO_{1}}{r} = \frac{PO_{1} - PB}{r} = \frac{PO_{1} - PS_{2}}{r} = \frac{C \times t_{O1} - C \times t_{2}}{r} = \frac{C \times \tau}{r}$$
(3)

Where c is the speed  $\tau = t_{SO1} - t_2$  of sound in the air.

Therefore, the included angle  $\varphi$  between the straight line  $P0_1$  from the sound source P and the positive direction of the axis X (here called the azimuth angle) can be expressed as (discussed in two cases):

When the next nearest point of the sound source P to disc 1 is the sensor  $S_3$  (taking Figure 3 as an example), it can be calculated according to formula (4):

$$\varphi = 45^{\circ} \times (subscript - 1) + \theta_1 \tag{4}$$

For example, in the case of disc 1 in Figure 3, the nearest sensor is  $S_2$ , and the next nearest point is the sensor  $S_3$ , then:

$$\varphi_{=}45^{0} \times (2-1) + \theta_{1} \tag{5}$$

When the next nearest point of the sound source P to disc 1 is the sensor  $S_1$  (taking Figure 3 as an example), it can be calculated according to formula (5):

$$\varphi_{=}45^{\circ} \times (subscript - 1) - \theta_{1} \tag{6}$$

Similarly, the azimuth angle from the sound source P to disc 2 can also be determined in this way.

#### (2) Determine linear equation

Slope  $k_1 = \tan \varphi$  of the line  $P0_1$  from the sound source P. Then the equation  $P0_1$  of the straight line can be expressed as equation (7):

$$y = k_1(x + D/2)$$
 (7)

Similarly, the equation of the straight line  $P0_2$  from the sound source P can be expressed as equation

$$y = k_2(x - D/2) \tag{8}$$

#### (3) Determine the coordinates of the impact point

The coordinate of the acoustic target P is the intersection point of equation (7) and equation (8), which can be solved by solving the equations:

$$\begin{cases} y = k_1(x + D/2) \\ y = k_2(x - D/2) \end{cases}$$

To solve this equation group:

$$x = \frac{D(k_1 + k_2)}{2(k_2 - k_1)} \tag{9}$$

$$y = k_1(x + D/2)$$
 (10)

It can be seen from the above derivation that the positioning error is mainly the error caused by time delay, the effective sound velocity error, and also related to the sensor position accuracy and the geometric shape and size of the acoustic array. When the array element layout is certain, the position accuracy can be improved through accurate measurement. The error of effective sound velocity can also be corrected according to the actual environmental factors. Therefore, only the directional and ranging errors caused by the range difference error caused by the time delay estimation error are analyzed. It can be seen from the expressions of x and y that the sources of errors are mainly azimuth  $\varphi$  and time delay errors  $\delta_{\tau}$ . In addition, the propagation speed c of sound in the air will also be affected by temperature and wind direction, and the installation accuracy of sensors and disks will also have a certain impact on the positioning accuracy. Azimuth angle  $\varphi$  is mainly determined by angle  $\theta$  calculation. Without considering other influencing factors such as time delay difference, the error of azimuth angle  $\varphi$  mainly comes from the error of angle  $\theta$ , and the angle  $\theta$  is

obtained by an approximate condition that is, the angle  $\angle O_1 BS_2$  is approximately regarded as a right angle. Therefore, the azimuth error  $\varphi$  can be determined as long as the error caused by this approximate condition is determined. The error caused by this approximate condition is determined by simulation. The Monte Carlo method is used for computer numerical calculation and simulation, that is, the real value  $\theta'$  of the angle  $\theta$  is calculated by taking a number of known sound source points, and the approximate value  $\theta'$  of the angle  $\theta$  is calculated by this approximate method. The size of the angle error  $\theta$  can be determined by comparing the difference between  $\theta$  and  $\theta'$ .

### 4. SIMULATION ANALYSIS

Let the polar coordinate  $(r,\theta)$  of the impact point be, which r and  $\theta$  are called the ranging distance and the azimuth angle. In simulation, the TDOA estimation error is 45  $\mu s$ , and the sound source area is 1 km  $\times$  1 km. The variation range is  $0 \le r \le 1000m$ . The variation range of azimuth angle  $\theta$  is  $0 \le \theta \le 180^\circ$ . Figure 4 shows the variation of ranging distance error and azimuth error with ranging distance.



Figure 4. Variation of ranging distance and azimuth error with ranging distance

In the simulation process, the variation range of azimuth angle  $\alpha$  is  $0 \le \alpha \le 180^{\circ}$ ; The variation range  $\beta$  of elevation angle is  $0 \le \beta \le 90^{\circ}$ ; The range of distance d variation is  $0 \le d \le 1000m$ ; Delay standard deviation  $\sigma_{\tau} = 25\mu s$ ; Sound speed c = 340m/s, L = 1 m,  $h_1 = 1 m$ ,  $h_2 = 2 m$ . The simulation results are shown in Figure 5, it can be seen that when the elevation angle  $\beta$  is close to  $90^{\circ}$ , the ranging error increases sharply, but the blind area is very small. Under the same delay error condition, the blind area is smaller than the ranging blind area of the five element stereo array, which has better ranging performance than the five element stereo array; When the azimuth  $\alpha$  is near to  $90^{\circ}$ , the ranging error is minimum, and when the azimuth  $\alpha$  is  $0^{\circ}$  and  $180^{\circ}$ , the ranging error is maximum. With the increase of distance, the ranging error also increases significantly.

In addition, the ranging error is also related to the position of the acoustic sensor  $S_5, S_6$ . Under the simulation conditions of L = 1m,  $h_1 = 1m$ , azimuth angle  $\alpha = 30^\circ$ , elevation  $\beta = 60^\circ$ , distance d = 500m, and delay errors  $\sigma_\tau = 25\mu s$ , the variation curve of the ranging standard deviation  $\sigma_d$  with the coordinate value  $S_6$  on the axis Z. At that time, the standard deviation  $\sigma_d = 337m$  of fixed distance, at this time, the error of  $S_5, S_6$  is maximum due to symmetry about the origin. It can also be seen from the curve that the distance error when  $h_2 > 1m$  is smaller than that when  $h_2 < 1m$ , so it is generally taken  $h_2 > h_1$  during array arrangement.



Figure 5. Distance standard deviation and elevation and azimuth curves

## 5. SUMMARY

The planar double disk array can effectively carry out long-distance positioning, and has the characteristics of convenient layout, simple model, and small amount of calculation, which is suitable for random range array. The method proposed in this paper is based on other positioning arrays. In order to improve the positioning performance, the positioning algorithm is derived, the positioning error is analyzed, and its positioning performance is effectively verified by simulation program.

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