# Nonlocal Photon Number States for Quantum Metrology

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## ABSTRACT

Quantum metrology utilizes nonclassical states (of light) to outperform the accuracy limits of its classical counterpart. We demonstrate the relevance of photon number Fock states and polarization entanglement for the experimental realization of interferometric quantum metrology applications.

**Keywords:** quantum entanglement, quantum metrology, quantum information, quantum physics

## 1. INTRODUCTION

One objective of metrology is to determine macroscopic (classical) observables such as position, momentum or interferometric phase with high precision. Obviously, the accuracy of the measurement outcome is restricted by the method used. Quantum metrology exploits specific quantum physical features such as entanglement[1] or non-classical states (of light)[2] to achieve a performance beyond the capabilities of a classical measurement device. Such "quantum advantage" has within recent years also been discovered in the field of quantum communication and quantum information-processing. There, quantum entanglement is utilized to achieve communication and computational tasks that outperform their classical counterparts. Well known examples include quantum state teleportation,[3,4] quantum dense coding,[5,6] quantum cryptography[7–10] and quantum computing.[11] Further examples are entanglement-assisted classical communication[12,13] to enhance the communication capacity in a noisy environment or methods to exploit the computational advantages provided by quantum entanglement for communication complexity problems.[14–16] Note that in all these quantum-enhanced schemes, entanglement is combined with specific projective measurements to achieve an advantageous performance over their classical analogues.

In the following, we will discuss the specific example of how to use path-entangled photon number states, so called "NOON- states", to achieve better than classical performance for interferometric measurements. In particular, we have utilized 2- and 4-photon states from spontaneous parametric down conversion (SPDC) to demonstrate pure 2- and 4-photon interferometry as a tool for quantum metrology.

## 2. PHOTON NUMBER STATES IN QUANTUM METROLOGY

Photon number "Fock"-states are nonclassical states of light[2] with a well-defined photon number per optical mode, that have direct applications in quantum metrology. For illustration, consider the situation in a Mach-Zehnder interferometer (see Figure 1a). There, single-photon interference occurs due to the superposition of two modes of propagation $a$ and $b$ for a single particle after entering the interferometer at the first beamsplitter. The path length difference $\Delta x$ induces a phase shift $\Delta\phi$ and gives rise to observable (single particle) interference in each of the two output modes $d_a$ and $d_b$ with detection probabilities $P_a \propto 1 + cos\Delta\phi$ and $P_b \propto 1 - cos\Delta\phi$, respectively. Therefore, the normalized intensity at the output port can serve as a direct measure for the optical path length difference (modulo the wavelength $\lambda$ of the probe photons) or also for measuring the position of the mirror M, if all other mechanical elements are considered fixed. In general, the sensitivity of this measurement is always ultimately limited by the noise characteristics of the probe state within the interferometer. For example, when using coherent states, i.e. a "classical" laser beam, as input to the interferometer, the phase sensitivity $\Delta\phi$ is eventually (shot noise) limited by the quantum phase fluctuations of the coherent probe state.[17] Since these fluctuations enter directly as measurement errors, $\Delta\phi$ decreases only with $1/\sqrt{<n>}$ when increasing the average photon number $< n >$ in the coherent state. In other words, when doubling the laser intensity, the measurement accuracy only increases by $\sqrt{2}$. Obviously, this situation is equivalent to sampling $< n >$ independent results of a true single particle interference experiment as described before.[18] This seems to be a non-optimal use of resources. In fact, the situation changes for the quantum metrology case when using N-photon states. By employing higher-order interference effects and consequently quantum entanglement, one can achieve a more optimal scaling with $\Delta\phi \propto 1/N$.

To see the origin of this advantage more clearly, we focus on the case of an N-particle Mach-Zehnder interferometer, in which the photons within the interferometer are in a superposition of being in either mode $a$ or $b$, which corresponds to a "NOON"-state of the form $\frac{1}{\sqrt{2}}(|N\rangle_a|0\rangle_b + e^{iN\phi}|0\rangle_a|N\rangle_b)$, where $|N\rangle_{a(b)}$ indicates the N-particle Fock state in spatial mode $a(b)$, $N = 0$ being an empty mode. In other words, the paths are entangled in photon number. Due to the fact that phase variations act on N photons simultaneously, the N-photon detection probability in each of the interferometer outputs is now given by $P_N \propto 1 + cosN\Delta\phi$, which corresponds to a N-fold reduction in the wavelength of the interference oscillations *. As a consequence, not only the resolution of the phase measurement is increased, but also the achievable accuracy $\Delta\phi$ demonstrates a stronger scaling behavior. When increasing the particle number $N$ in the NOON-state, the phase sensitivity increases with $1/N$, which is a "quantum gain" of $1/\sqrt{N}$ compared to the classical case of coherent states. When doubling the photon number, the accuracy of the measurement is also doubled.

Other interferometric quantum metrology methods that do not rely on NOON states can also achieve this enhanced scaling behavior. Prominent examples are number-state based correlated input port interferometry (with separable or entangled inputs)[21–24] or squeezed coherent state interferometry.[25,26]

A simple understanding of this scaling behavior can be obtained from the "Dirac-relation" $\Delta N \propto 1/\Delta\phi$ between the photon-number uncertainty $\Delta N$ and the phase uncertainty $\Delta\phi$ in a light field[27] †. Applied to the state propagating through the interferometer: a quantum state (of light) with maximal relative number uncertainty $\Delta N$ between the modes will obviously minimize its inherent fluctuations $\Delta\phi$ in phase difference and is thus optimally suited for interferometric metrology measurements of the above type. This is obviously fulfilled by NOON-states. Specifically, the NOON-state's relative photon number uncertainty is $\Delta N = N$, which yields the wanted scaling behavior $\Delta\phi \propto 1/N$. In contrast, a coherent state entering the interferometer with a mean photon number $< n >$ will retain its photon statistics, which asymptotically results in relative number fluctuations $\Delta N \propto \sqrt{n}$ and thus $\Delta\phi \propto 1/\sqrt{n}$. For other input states, an explicit calculation of the relative number uncertainty $(\Delta N)^2 =< N^2 > - < N >^2$ (with $N = N_a - N_b$) of the state propagating within the interferometer can be performed to investigate the scaling behavior and thus to test for a quantum advantage of the probe state used. For example, in the case of correlated input port interferometry, the initial input state entering the two input ports $A$ and $B$ of the first Mach-Zehnder beamsplitter is $|N/2, N/2\rangle_{A,B}$. It is a straightforward calculation to show the asymptotic scaling $\Delta N \propto N$ necessary to achieve the wanted quantum metrology phase sensitivity. Although this sensitivity is typically referred to as the Heisenberg-limit due to its relation to quantum noise fluctuations, it can be seen from above that, for the photon case, it is helpful to also bear in mind the Dirac-relation.

To make optimal use of photon number states for quantum metrology, state engineering is necessary to achieve states that propagate within the interferometer with minimum relative phase uncertainty $\Delta\phi$ (and therefore maximum relative number uncertainty $\Delta N$). The most obvious example being the NOON-state with large photon number N. The special case of $N = 2$ has already been realized in several experiments.[28–30] It was commonly believed that the realization for states with $N > 2$ requires the use of non-linear gates[31] or additional "ancilla" detectors with single-photon resolution.[32] Unfortunately, each of these schemes is not feasible with current technologies. Only recently, this limit has been overcome by two different approaches based on linear

*It has therefore been suggested[19] to attribute an effective de Broglie wavelength λ/N to the quantum state, in analogy to interference with large molecules consisting of N atoms.[20]

†For the case of a photon number state, $\Delta N = 0$ and the phase is thus maximally undefined.

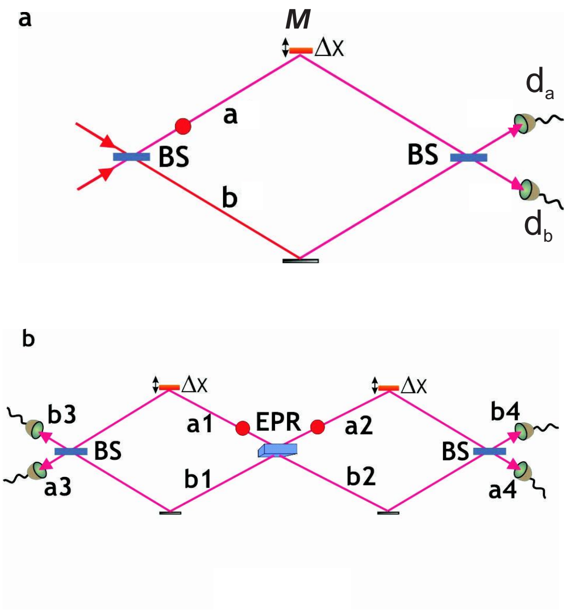


**Figure 1.** Schematic drawing of a single-photon Mach-Zehnder interferometer (a) and a two- and four-particle interferometer (b). The phases are changed by varying the path-length via the position $\Delta x$ of the mirror M. In the Mach-Zehnder case, single-photon interference occurs due to superposition of two possible modes of propagation a1 and b1 for a single particle entering the interferometer at the first beamsplitter (BS). Two-photon interference can be achieved by using the emission of time-correlated photon pairs from a source (EPR) into spatially separated pairs of modes a1-a2 or b1-b2. A pair wise recombination of beams a and b results in maximally path-entangled two-photon states, which lead to perfect two-photon interference fringes when varying the length of the paths $\Delta x$.

optics.[33–35] In the following, we will present one of the schemes[33] and demonstrate how to utilize higher photon number states for quantum metrology for the specific example of $N = 4$.

## 3. EXPERIMENTAL 4-PHOTON NOON STATES WITH LINEAR OPTICS

Our approach is based on separating photon pairs into different pairs of modes and using nonlocal polarization correlations rather than distinguishing photon numbers or employing nonlinear beamsplitters. We allow 4 photons to propagate along two spatially separated pairs of modes, where in the ideal case each mode is occupied by two photons. This is in analogy to the original proposal for two-particle interferometry by Horne, Shimony and Zeilinger,[36] where one pair of particles is spatially separated into two pairs of modes to generate intrinsic two-particle entanglement in the form of two-photon NOON-states. In their case, two-particle interference can be observed by pair-wise recombining the beams interferometrically (see Figure 1b) and measuring the two photons simultaneously. The observed interference[37] is due to the fact that after the beamsplitter one cannot distinguish which path (here: pair of modes) was taken by the two-photon object. Similarly in our case, when four photons are distributed over two pairs of modes one would expect 4-photon interference if the photons overlap at the beamsplitters in such a way that no information of their path can be obtained. The required measurement is then a 4-fold coincidence detection in the four output modes a3, a4, b3 and b4. In contrast to other proposals, we do not need additional beamsplitters and/or detectors with single-photon resolution.[38]

To achieve this goal, we exploit type-II SPDC.[39] An ultra-violet pulse passes through a beta-barium-borate (BBO) crystal, probabilistically producing pairs of energy-degenerate polarization-entangled photons into the spatial modes a1 and a2 (see Figure 2), described by the Hamiltonian $H_{\Phi^+} \propto (a^\dagger_{1H} a^\dagger_{2H} + a^\dagger_{1V} a^\dagger_{2V})$. The pulse is then reflected back through the crystal with the same interaction but producing a state described by $H_{\Psi^+} \propto (a^\dagger_{1H} a^\dagger_{2V} + a^\dagger_{1V} a^\dagger_{2H})$. This means that the setup is aligned to produce the maximally entangled state $\Phi^+ = 1/\sqrt{2}(|HH\rangle_{a1a2} + |VV\rangle_{a1a2})$ for each of the pairs generated into the pairs of modes a1-a2 and the state

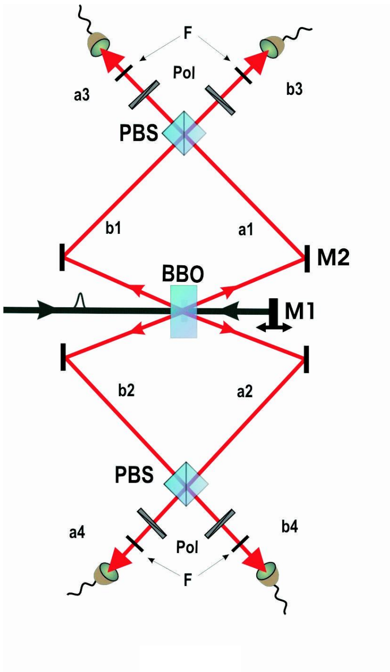


**Figure 2.** Experimental setup for a one-, two- and four-photon interferometer (from[33]). A UV-pulse passes through a beta-barium borate (BBO) crystal twice, each time generating a frequency-degenerate, polarization-entangled photon pair with probability $p$, and two pairs with probability $p^2$. 3nm bandwidth filters (F) in front of each detector assure the overlap of the photon-wavepackets at the polarizing beamsplitters (PBS). The UV-pump is reflected by mirror M1, which is mounted on a computer-controlled translation stage. By scanning the position $\Delta x$ of M1 with a step size of 1 $\mu m$ and performing fine adjustment of the position of M2, we achieved the temporal overlap of modes a1 and b1, and of modes a2 and b2. An additional piezo translation stage is used to move the mirror M1 and to perform a fine scan around the region of the best overlap. Detecting 4-photon coincidences behind a 45° polarizer (Pol) while varying M1 leads to the observed interference fringes.

$\Psi^+ = 1/\sqrt{2}(|HV\rangle_{a1a2} + |VH\rangle_{a1a2})$ into the pairs of modes a1-a2. Here H and V indicate horizontal and vertical polarization of the photon.

We first consider the case where only one pair of entangled photons is emitted on a double pass of the UV pulse through the crystal. There are two probability amplitudes which will contribute to the emerging two-photon state, i.e. either the pair is emitted into the pair of modes a1-a2 or into the pair of modes b1-b2, resulting in the two-photon NOON-state $|\Psi\rangle \propto (|2\rangle_{a1a2}|0\rangle_{b1b2} + e^{i2\Delta\phi}|0\rangle_{a1a2}|2\rangle_{b1b2})$. This scheme can be generalized to the case of four photons and even arbitrary photon numbers, if we take into account higher orders of the emission process. Consider the case where two pairs of photons are emitted on a double pass of the pump beam through the crystal. There are three possibilities which will equally contribute to the overall 4-photon state: (1) a 4-photon state can be emitted into the mode pair a1-a2 via the process $H_{\Phi^+}$, (2) a 4-photon state can be emitted into the mode pair b1-b2 via the process $H_{\Psi^+}$, and (3) two pairs are emitted into the pair of modes a1-a2 and b1-b2. The overall 4-photon state is therefore given by $|\Psi\rangle \propto (|4\rangle_{a1a2}|0\rangle_{b1b2} + e^{i4\Delta\phi}|0\rangle_{a1a2}|4\rangle_{b1b2} + e^{i2\Delta\phi}|2\rangle_{a1a2}|2\rangle_{b1b2})$. Finally, in order to suppress the unwanted contributions $|2\rangle_{a1a2}|2\rangle_{b1b2}$, we exploit the nonlocal polarization correlations of the prepared state. Explicitly, we use two polarizing beamsplitters to perform a bilateral parity check on the polarization[40]: for the $\Phi^+$-Bell state both photons are always transmitted or reflected, while for the $\Psi^+$-Bell state one photon is always transmitted and one is reflected, such that no interfering amplitudes for two-fold detection events can build up. Only a double-pair emission on each side, where a four-photon is emitted either forwards or backwards, contributes to the 4-photon state after the PBS and gives rise to pure four-photon interference. Consequently, the overall state contributing to our detection signal can be written as

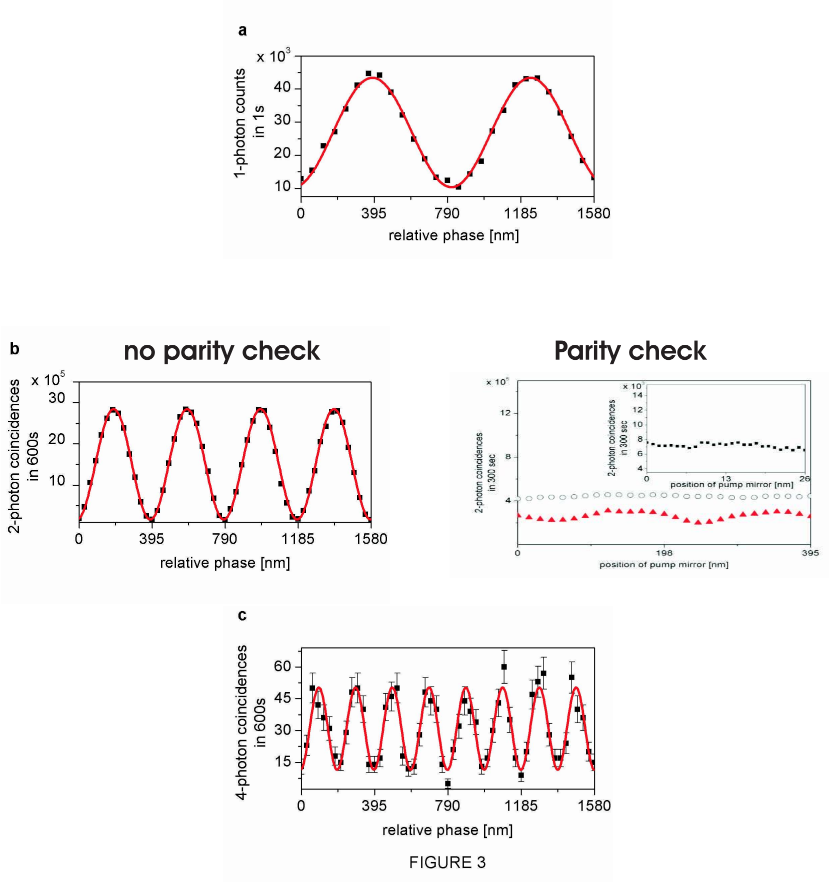


**Figure 3.** Experimental demonstration of one-(a), two-(b) and four- photon (c) NOON-state interferometry (see text and also[33]).

$|\Psi\rangle_{parity} \propto (|4\rangle_{a1a2}|0\rangle_{b1b2} + e^{i4\Delta\phi}|0\rangle_{a1a2}|4\rangle_{b1b2})$, which is the wanted 4-photon NOON state.

Figure 3 compares the observed 4-photon interference effect (Figure 3c) with the single-photon interference (Figure 3a) as were obtained with the same setup. Note that the parity check also suppresses all 2-photon interference contributions. For comparison, two-photon interference data obtained with (Figure 3b right) and without (Figure 3b left) parity check is also shown. Fits to the data reveal a subsequent reduction of the oscillation wavelength from $823 \pm 46$ $nm$ for the single-photon case over $395 \pm 16$ $nm$ for the 2-photon case and $194 \pm 9$ $nm$ for the 4-photon case. This corresponds to a deviation of 4% from the expected halving of the wavelength for single-particle to 2-particle interference and of 2% for 2-particle to 4-particle interference. The deviation is within the limits given by the thermal long-term stability of our interferometric setup.

Our four-photon state has the additional interesting property that it is nonlocal as it is a superposition of four photons either in mode a1 and a2 or b1 and b2. In principle, this can be extended to higher-order interference effects because, obviously, when more than two double-pairs are emitted from the crystal the suppression of all lower-order interferences can be achieved by a proper projection analogous to the N=4 case.

## 4. CONCLUSION

To conclude, we have demonstrated how SPDC and polarization entanglement can be utilized to create nonlocal photon number "NOON" states, which are important for quantum metrology applications. From a more general point of view, one may ask whether quantum metrology would also yield a thermodynamic advantage in the context of measurements. As was demonstrated for the example of a Mach-Zehnder interferometer, quantum metrology allows to use less resources (i.e. photons within the probe state) to obtain the same amount of information of a physical system (i.e. mirror position with a certain accuracy). This might provide novel insights also for measurements on quantum states and quantum heat engines. Possibly, quantum metrology methods are required for such devices to achieve optimal performance.

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