NEQ: its progenitors and progeny

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ABSTRACT

The historical evolution of the concept of noise-equivalent quanta (NEQ) and its application to task-based assessment of image quality is surveyed, with particular emphasis on the seminal contributions of Robert F. Wagner.

Keywords: Fourier analysis, image quality, noise-equivalent quanta

1. INTRODUCTION

This paper is about three great melodies that wafted through the halls of optics and imaging during the last half-century -- and about one great scientist who danced nimbly to all three. The three great melodies are Fourier optics, statistical optics and image quality, and the scientist, whose career we commemorate in this symposium, is Robert F. Wagner. It is the purpose of this paper to survey that career from the standpoint of a single mathematical concept, noise-equivalent quanta or NEQ.

We begin in Sec. 2 with a brief and selective historical overview of Fourier analysis and its evolution as a tool for image analysis and evaluation. We look similarly at statistical optics and how its critical role in image quality emerged, and we look at some key mileposts along the path towards a modern theory of image quality itself.

Sec. 3 traces the origins of NEQ and the related concept of DQE (detective quantum efficiency), discussing explicitly the key role played by Bob Wagner in bringing together diverse mathematical viewpoints. Sec. 4 briefly surveys the many generalizations of basic NEQ theory that have emerged since Bob's seminal contributions in the 1970s, and Sec. 5 lists some further opportunities for exploitation of his insights. An unconventional summary is presented in Sec. 6.

2. HISTORICAL OVERVIEW

2.1 Fourier optics

Fourier analysis was developed, and accepted as a scientific tool, in the nineteenth century, and before the century was out, it was being applied to optics and imaging. Fraunhofer, Rayleigh and Sommerfeld, among others, applied Fourier analysis to diffraction. Indeed, in the Fraunhofer approximation, a diffraction pattern *is* a Fourier transform.

The first direct application of Fourier theory to image formation was carried out by Ernst Abbé in 1873, when he developed a diffraction-based theory of microscopes [1]. In addition to his scientific contributions, Abbé, who was the director of the Carl Zeiss Laboratory, also introduced many important social innovations, including the eight-hour workday and profit-sharing with his employees.

The concept of an optical transfer function for incoherent imaging was published in French by Duffieux [2] in1946, but it did not receive much attention in the western literature until 1959, when it was discussed in some detail by Born and Wolf in *Principles of Optics* [3].

In 1956, Ronald Bracewell [4] recognized the importance of Fourier theory in radioastronomy, and by extension to all of tomography, and in 1962 Emmett Leith and Juris Upatnieks brought a Fourier viewpoint from communications theory into optics and in particular to holography [5]. Nevertheless, much of the optics community was slow to adopt Fourier methods. As late as 1968, SPIE held a largely tutorial Seminar-in-Depth on the modulation transfer function and its uses [6].

Medical Imaging 2009: Image Perception, Observer Performance, and Technology Assessment,
edited by Berkman Sahiner, David J. Manning, Proc. of SPIE Vol. 7263, 72630F

2.2 Statistical optics

That optics and imaging are inherently statistical derives mainly from the statistical nature of the photoelectric effect, recognized by Einstein in 1905. Later work revealed that photoelectrons are almost always well described by Poisson statistics, the Bosonic character of photons not withstanding.

Application of these statistical ideas to imaging originated with the work of Albert Rose [7] and his contemporary and colleague, Otto Schade [8]. Rose developed a matched-filter approach, now known as the Rose model, which explained many aspects of signal perceptibility in noisy images. Schade extended Rose's work and in 1964 developed a concept called the Detail Signal-to-Noise Ratio, which is an important starting point in our NEQ story.

The influence of statistics on imagery was well established by the early sixties, especially with the publication of a book chapter by Dennis Gabor (the inventor of holography) on Light and Information [9] and the book by Edward O'Neill, Introduction to Statistical Optics [10].

2.3 Image quality

The beginnings of a modern theory of image quality can be traced to an often-cited 1956 paper by Fellgett and Linfoot on the assessment of optical images [11]. In one sense, however, this paper may have delayed the development of a rigorous theory of image quality because it subscribed to the notion that the purpose of imaging was to reproduce the object as faithfully as possible. Whether fidelity is defined in terms of mean-square error or by various information measures, however, we now realize that faithful reproduction is a fundamental impossibility. A real-world object (as opposed to a computer simulation) is a function, described by an infinite set of parameters, and real-world imaging systems (as opposed to mathematical idealizations) collect only a finite set of measurements, so any image conveys only an infinitesimal fraction of the information about the object, even in the absence of noise. Stated differently, all real systems have null functions, so many different objects can produce exactly the same mean image. To which should we be faithful?

An important break with the fidelity paradigm was made by Horace Barlow in 1962 [12]. His introduction of the concept of an ideal observer in vision research not only acknowledged that the purpose of an image is to help the observer make decisions about the object, rather than reproducing it in its entirety, but it also proposed the optimum way of making these decisions. When the images are noisy, as all are, this leads us into the realm of statistical decision theory.

In 1964, J. L. Harris considered the problem of distinguishing between two known objects, for example one consisting of a single point source and the other consisting of two closely spaced, mutually incoherent point sources, each of half the intensity of the single source [13]. This discrimination problem is known in the literature as the Rayleigh task because it was suggested by Lord Rayleigh as a way of defining the resolution of an imaging system. Rather than appealing to the familiar Rayleigh criterion for this task, however, Harris consider the fundamental limits on the problem arising from system transfer characteristics and the noise in the image. With this step, therefore, Harris connected image quality with a specific task and with statistical decision theory.

Bob Wagner was greatly influenced by the work of Barlow and Harris, and in a 1972 paper [14] entitled "An assortment of image quality indices-- Can they be resolved?", he wrote "The question of image quality has been an elusive one to define. One thing , however, is *certain* -- that it *must* be defined in terms of the task the image is destined to perform" [emphasis added].

Wagner's certitude, that image quality is defined fundamentally in terms of task performance, has been accepted in some branches of imaging but widely ignored in others. In radiology and nuclear medicine, there is a long history of research in task-based assessment, beginning with the pioneering work of Russell Morgan, Kurt Rossman and Bob Beck, followed soon by important contributions from Ben Tsui, Kunio Doi, Charles Metz and many others. In other areas of imaging, however, Wagner's strong assertion is scarcely acknowledged, and workers in these fields cling to fundamentally flawed metrics such as pixel signal-to-noise ratio (SNR) and contrast-to-noise ratio (CNR), which have no clear relation to the purpose for which the image was produced.

3. ORIGINS AND THEORY OF DQE AND NEQ

The immediate progenitor of the NEQ concept was the detective quantum efficiency (DQE), first proposed for a nonimaging optical detector and then for photographic film. For a single-element optical or infrared detector viewing either some background illumination or the background plus a signal, one can define a simple SNR by the difference in the mean detector outputs with and without the source divided by the square root of the average variance of the noise on the detector output. Denoting this ratio as SNR_{out}, we can define the *DQE* as

(1) $$DQE = \frac{SNR_{out}^2}{SNR_{in}^2},$$

where SNR_{in} is the input SNR, that which would be achieved by an ideal, photon-counting detector.

Similar definitions of DQE in the context of photography were proposed in the late 1950s by Fellgett [15] and Jones [16], but the work that had considerable influence on Bob Wagner was the 1963 paper by Rodney Shaw, The Equivalent Quantum Efficiency of the Photographic Process [17]. The ideas in that paper were further expounded by Chris Dainty and Shaw in the landmark book, Principles of Image Science, published in 1974 [18].

By 1978, Wagner had achieved a full synthesis of the ideas of Schade, Harris and Shaw, and he presented them in a paper entitled Decision Theory and the Detail SNR of Otto Schade [19]. In it, he definitively answered his own 1972 question: the myriad metrics of image quality could indeed be resolved into a unified and practical theory.

Key to the resolution was a Fourier-domain expression for NEQ, which we can best appreciate by considering the task of discriminating between two known signals in stationary, Gaussian noise. This task is often referred to as *signal known exactly* (SKE) because the two signals are always of the same, nonrandom form and always known to the observer; the only randomness is which one is present.

For the SKE discrimination task in Gaussian noise, the ideal observer is a prewhitening matched filter, which computes a test statistic $t(\mathbf{g})$, expressed in the 2D Fourier domain as

(2) $$t(\mathbf{g}) = \int d^2\rho \frac{\Delta S^*(\boldsymbol{\rho})G(\boldsymbol{\rho})}{NPS(\boldsymbol{\rho})} \quad ,$$

where $\boldsymbol{\rho}$ is the 2D spatial frequency vector, $G(\boldsymbol{\rho})$ is the 2D Fourier transform of the noisy image, $\Delta S(\boldsymbol{\rho})$ is the difference between the transforms of the two signals to be discriminated (asterisk denoting complex conjugate), and $NPS(\boldsymbol{\rho})$ is the noise power spectrum. Making the decision by comparing the test statistic of (2) to a varying threshold generates the well-known ROC (receiver operating characteristic) curve, and the ideal observer is the one that maximizes the area under the ROC curve or AUC, a well-established metric of image quality.

The performance of the ideal observer on this SKE task can also be specified rigorously by a new kind of signal-to-noise ratio, often referred to as d' or d_a. This quantity is defined analogously to the SNR used for DQE in (1) but instead of involving the *signal* means and variances, it is the difference between the mean values *of the test statistic* divided by the square root of its average variance. It can be shown that this SNR is sufficient to determine the AUC if $t(\mathbf{g})$ is a Gaussian random variable, as it will be for Gaussian noise.

The ideal-observer SNR for an SKE task can be expressed in the Fourier domain as

(3) $$SNR_{ideal}^2 = \int d^2\rho \frac{|\Delta S(\boldsymbol{\rho})|^2}{NPS(\boldsymbol{\rho})} \quad .$$

For a linear, shift-invariant imaging system, the Fourier transform of the difference between the two image signals is related to the Fourier transform of the difference between the two objects to be discriminated by

(4) $$\Delta S(\boldsymbol{\rho}) = \Delta F(\boldsymbol{\rho}) TF(\boldsymbol{\rho}) ,$$

where $TF(\boldsymbol{\rho})$ is the system transfer function. Thus the ideal-observer SNR for this problem is given by

(5)
$$SNR^2_{ideal} = \int d^2\rho \frac{|\Delta S(\boldsymbol{\rho})|^2}{NPS(\boldsymbol{\rho})} = \int d^2\rho \frac{|TF(\boldsymbol{\rho})|^2}{NPS(\boldsymbol{\rho})} |\Delta F(\boldsymbol{\rho})|^2$$
$$= |TF(0)|^2 \int d^2\rho \frac{|MTF(\boldsymbol{\rho})|^2}{NPS(\boldsymbol{\rho})} |\Delta F(\boldsymbol{\rho})|^2 \equiv |TF(0)|^2 \int d^2\rho \, NEQ(\boldsymbol{\rho}) \, |\Delta F(\boldsymbol{\rho})|^2 ,$$

where $MTF(\boldsymbol{\rho})$ is the modulation transfer function, defined as $|TF(\boldsymbol{\rho})|/|TF(0)|$.

In summary, the performance of the ideal observer for discriminating between two known signals in stationary Gaussian noise with a linear, shift-invariant imaging system is given by

(6)
$$SNR^2_{ideal} = |TF(0)|^2 \int d^2\rho \, NEQ(\boldsymbol{\rho}) \, |\Delta F(\boldsymbol{\rho})|^2 ,$$

where

(7)
$$NEQ(\boldsymbol{\rho}) = \frac{|MTF(\boldsymbol{\rho})|^2}{NPS(\boldsymbol{\rho})} .$$

Thus, NEQ provides the ideal-observer figure of merit (FOM) for discrimination between two known signals in Gaussian noise. This FOM is in the form of an easily evaluated 2D integral. Moreover, the NEQ formalism for this specific case leads to a neat factorization of the integrand, into one factor, $NEQ(\boldsymbol{\rho})$, describing the system properties and another factor, $|\Delta F(\boldsymbol{\rho})|^2$, describing the task; the system can be optimized simultaneously for all tasks by maximizing the NEQ, though only for SKE detection with linear, shift-invariant systems and stationary Gaussian noise.

Curiously, even though the very term noise-equivalent *quanta* implies a connection with Poisson counting statistics, the NEQ expression for SNR is rigorously related to the ideal-observer performance only for Gaussian noise. The corresponding SNR expression for Poisson noise was formulated by Cunningham *et al*. [20] and applied to radiological problems in a 1981 paper by Wagner, Brown and Metz [21]. In practice, however, the expressions in this section are usually excellent approximations for Poisson or mixed Poisson/Gaussian noise.

4. GENERALIZATIONS

As noted, the theory in the last section is limited to a certain task and certain assumptions about the imaging system and the noise. Generalizations of the theory are needed because the image noise is never strictly stationary and the noise processes are more complicated than the additive, Gaussian model. Moreover, digital images are discrete sets of numbers rather than functions of continuous variables, so the Fourier integral is not strictly applicable. In addition, interesting imaging systems are never strictly shift-invariant, and in fact they may not even conserve spatial dimensionality; for example, they may map a 3D object to a 2D image. The systems may not be linear, even approximately, and the task may not be one that is performed optimally by linear observers like the prewhitening matched filter.

Numerous refinements and generalizations of the NEQ concept have appeared in the three decades since Wagner's 1978 paper. The first, a joint effort [22] between Bob and the author of this paper, was to break away from the paradigm of known, nonrandom objects by supposing that a signal, if present was superimposed on a random background representing structure or clutter in the object. If the background was modeled as a stationary random process, the resulting SNR was still factorizable as in (6), but with an NEQ given by

(8)
$$NEQ(\boldsymbol{\rho}) = \frac{|MTF(\boldsymbol{\rho})|^2}{NPS_{meas}(\boldsymbol{\rho}) + NPS_{obj}(\boldsymbol{\rho})\,|TF(\boldsymbol{\rho})|^2}$$

where $NPS_{obj}(\boldsymbol{\rho})$ is the noise power spectrum of the background object. Note that the power spectrum for the object is filtered by the square of the transfer function.

Another generalization, by Don Wilson [23, 24], was to break away from the requirements of stationarity and shift-invariance by considering detection of a known signal at a specific location $\mathbf{r}_0$ in the image. If it could be assumed that the system point-spread function and the noise statistics were slowly varying in the vicinity of the signal location, it was possible to define local MTfs and local power spectra specific to the signal location $\mathbf{r}_0$, and the resulting SNR expression was in the form of (6) but with a local NEQ given by

(9)
$$LNEQ(\boldsymbol{\rho};\mathbf{r}_0) = \frac{|\, LMTF(\boldsymbol{\rho};\mathbf{r}_0)\,|^2}{LNPS_{meas}(\boldsymbol{\rho};\mathbf{r}_0) + LNPS_{obj}(\boldsymbol{\rho};\mathbf{r}_0)\,|\, LTF(\boldsymbol{\rho};\mathbf{r}_0)\,|^2} \quad ,$$

with the prefix L denoting local quantities.

The prewhitening matched filter with which we began this discussion is a linear filter that implements the ideal test statistic for SKE detection in stationary, Gaussian noise. Further elaborations of the theory led to the formulation of a ideal linear observer, called the Hotelling observer, which maximizes an SNR defined generally as the difference between the mean values of the test statistic divided by the square root of its average variance. For a discrete random image described by a vector $\mathbf{g}$, the resulting optimized SNR has the form [24],

(10)
$$SNR_{Hot}^2 = \Delta\overline{\mathbf{g}}^t \mathbf{K}^{-1} \Delta\overline{\mathbf{g}} \quad ,$$

where $\Delta\overline{\mathbf{g}}$ is the difference in mean vectors for the two hypotheses to be discriminated (*e.g.*, signal present and signal absent) and $\mathbf{K}$ is the average covariance matrix of $\mathbf{g}$. This formula makes no assumptions of statistical stationarity or shift invariance, and therefore it is more difficult to compute than the NEQ expression in (6), but it covers a much wider range of problems. For example, it can incorporate random signals, random backgrounds and arbitrary noise covariance. An explicit analogy with (9) is obtained by decomposing the image covariance matrix into components arising from measurement noise and object variability, with the result in the form [24, 25]

(11)
$$SNR_{Hot}^2 = \Delta\overline{\mathbf{g}}^t [\mathbf{K}_{meas} + \mathbf{H}\ \mathbf{K}_{obj} \mathbf{H}^{\dagger}]^{-1} \Delta\overline{\mathbf{g}} \,,$$

where $\mathcal{H}$ is an operator describing the system and the dagger denotes its adjoint.

Many further generalizations of the NEQ concept have also been developed, all inspired by Bob Wagner's 1978 paper and many with his direct involvement. One such is the Fourier crosstalk matrix [26], which is an exact description of *any* linear system that produces discrete images from an object of bounded support. Relying on Fourier series expansions rather than Fourier integrals, the Fourier-crosstalk approach produces NEQ-like SNR expressions even for systems with no semblance of shift-invariance. In one paper, it was described as "Fourier analysis where you would least expect to find it" [27].

The extension of NEQ to a nonlinear system was achieved by Zemp and Abbey [28] with their generalized NEQ for ultrasound, and the extension of nonlinear observers and nonlinear tasks was begun in an early Wagner SPIE paper [29]. The extension to radiographic systems that count photons and provide energy information was by Tapiovaara and Wagner [30].

5. OPPORTUNITIES

In spite of all of these extensions and elaborations of the basic NEQ concept, many opportunities for further research remain. Any of the NEQ-based FOMs can be used to optimize imaging systems for best task performance. It is even possible to adapt a system in real time to optimize its performance for a specific patient rather than for a class or ensemble of patients [31].

Further research is needed to account for randomness in the imaging systems itself and to allow both object and system to vary with time [32]. The most general problem is to optimize a random, dynamic imaging system for imaging random, dynamic object, with the optimization to be based, of course on task performance. New statistical models will be required for this enterprise.

An important goal, enunciated often by Bob Wagner, is to "impact the culture", to spread the message of task-based assessment and optimization throughout the medical imaging community and to the broader world of imaging. Doing so will require standardized methodologies and software.

6. SUMMARY AND CONCLUSIONS

We can summarize the impact of Bob Wagner's NEQ studies with an ancient Chinese poem:

> All that is spatial has a sound.
> Tone emerges from harmony.
> Harmony emerges from concord.
> Harmony and concord are the roots
> from which music, laid down by the ancient kings, emerged.
>
> Spring and Autumn by Lü Pu Wei,
> 3rd century B.C., vol. III/5

Achieving concord -- harmonization of scientific ideas and consensus among scientists -- was a major objective of Bob Wagner's career. This paper has illustrated the enormous progress towards this goal in the realm of image quality that was achieved during his lifetime, and through his guidance and inspiration. We can all honor his work and his memory by striving ever more for concord.

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