Quantum Computing with Analog Circuits: Hilbert Space Computing

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ABSTRACT

We first point out the practical difficulties of *universal* quantum computing which may prohibit practical applications as *universal computers*. Then we show how to apply analog microelectronic circuits to realize the architecture, data processing and parallel computing abilities of quantum computing via Hilbert space computing with analog circuits. Such a Hilbert-space-analog (HSA) computer simulates the Hilbert space and its operators, and it is able to use and test quantum algorithms developed for the real quantum computers. Such a computer would be free of most of the practical difficulties of realizing and running a real quantum computer. This computer can be made universal. It is remarkable that by using the same numbers of transistors as in today's PCs, such a HSA computer can manipulate $\sim 10^7$ analog numbers corresponding to ~23 qubits, simultaneously, by quantum-parallel processing.

Keywords: parallel computing, future of quantum computing, analog computing

1. INTRODUCTION

It has been realized[1-3] that the computational operation called quantum computing does not need quantum systems. A quantum computer[4] with maximal capacity would use the information in the following superposition of states

$$\psi_{total}(x,t) = \sum_{\substack{i=1 \\ k=0,1}}^{i=2^N} a_i \prod_{j=1}^{N} \psi_j^{(k)}(x,t) \qquad , \tag{1}$$

where N is the number of quantum bits (qubits) in the system; $\psi_j^{(k)}(x,t)$ is a single qubit wavefunction which belongs to the j^{th} qubit; k is a digital variable where zero corresponds to the "down" state and one corresponds to the "up" state when we have spins; and a_i is the weight of the i^{th} superposition term. In the case of 2 qubits, this superposition is as follows:

$$\psi_{total}(x,t) = a_1\psi_1^{(1)}(x,t)\psi_2^{(1)}(x,t) + a_2\psi_1^{(0)}(x,t)\psi_2^{(0)}(x,t) + a_3\psi_1^{(1)}(x,t)\psi_2^{(0)}(x,t) + a_4\psi_1^{(0)}(x,t)\psi_2^{(1)}(x,t) \tag{2}$$

Concerning data handling, the information is in the square of the absolute value of the coefficients a_i. In a quantum system, if we have N proper single particle states (what we call qubits here), all the $|a_i|^2$ values can represent different numbers. The only relation which has a constrain on these 2^N piece of numbers is the normalization relation:

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$$|a_1|^2 + |a_2|^2 + ... + |a_{2^N - 1}|^2 + |a_{2^N}|^2 = 1 \quad , \qquad (3)$$

so basically 2^N-1 numbers can be set up independently. Therefore, quantum information represents a large size memory with a relatively small number of memory elements (bits). It is important to note here that these numbers are inherently analog numbers, not digital ones. When we execute a quantum process on these qubits, this operation will act on all the 2^N elements in Eq. 1, so it will influence all the 2^N analog numbers. This is the reason why expectations for quantum computing have skyrocketed recently. The apparent huge parallelism is very appealing and it suggests extraordinary speed. However, in Section 2, we will show that there is no "free lunch". Though qubits need only a small number of quantum objects, the parallel manipulation of the numbers needs at least $N*2^N$ objects in a universally programmable computer. This fact raises the question discussed in this paper: *Can all these elements and the "qubits" be classical physical objects, more specifically analog circuits, because these numbers are analog numbers?* As we will show, the answer is yes.

From a mathematical point of view, quantum computing with N qubits is equivalent to unitary operations on 2^N vectors in the Hilbert space and the apparent parallel computational potential is called quantum parallelism. The Hilbert space and these operations can be realized in proper classical physical systems, too. Therefore, it is more proper to call "quantum computing" *Hilbert space computing* (HSC) which can be done either in quantum or classical physical systems. Quantum computing is nothing else than HSC realized in quantum systems. The main known advantage of quantum computing is the parallelism and it is important to emphasize that this inherent property of the Hilbert space exists also in those classical physical systems which can be described by Hilbert space.

The quantum physical realization of HSC is probably not the best representation for best computing performance with reasonable economical efforts, as it is pointed out in Section 2. Quantum computing may be only a historical step to get to the idea of HSC. In the rest of this section, we survey the previous initiatives for classical physical HSC .

Ferry and coworkers [1], who are the first authors to propose classical physical systems for HSC, describe an electromagnetic wave-based HSC. As an introductory example, they show an optical grid which makes the Fourier transform of the amplitude distribution of the incoming beam of light along its cross section. The observation angle of the output wave will be the output Fourier-variable ("generalized frequency"). This system is a HSC unit and it can be built up so that in corresponds to quantum Fourier transformation. The Fourier transformation is obtained in one calculation step, so it is as massively parallel computation as quantum Fourier transformation.

O'uchi and coworkers [2] are the first authors who have built a real classical physical HSC. This computer is working with digital circuitry. The parallel digital circuitry built from standard logic integrated circuits elements models the 8-qubit Hilbert space and unitary transformations in it.

2. PRACTICAL DIFFICULTIES WITH UNIVERSAL QUANTUM COMPUTING

To appreciate the classical implementations of HSC, in this section we focus on the problematic sides of quantum computers.

A large body of articles and media features has been published about the positive side of the quantum computation idea. To achieve an effective progression in research and technology, it is also necessary to consider the adverse characteristics. Here we consider the most apparent problems and limits of universal quantum computing. We define "universal quantum computing" in the following way: a quantum computer that has the full capacity offered by quantum information and that can be programmed to solve arbitrary tasks allowed by the properties of quantum information. Less–then–universal quantum computers would have less problems, but also a less impressive performance. Special-purpose quantum computers may use large number of qubits but may not use the full power of quantum parallelism; they have fewer problems but also less computational power.

i) Complexity and size. *Mass*(ive) problem.

First, let us consider the complexity of the quantum gate system of universal quantum computation, which sets a limit on the achievable number of useful qubits. Any reasonably useful quantum computing application, either quantum or classical Hilbert space computation, needs large-scale integration of elements because of the exponentially growing number of quantum gates versus the number of qubits [4].

More precisely, for a universal Hilbert space computer, either quantum or classical, the minimal number of quantum gates is 2^N because at least one quantum gate has to act on each of the 2^N elements of the superposition. However, the quantum gates are not single elements; they contain elements which we here call acting elements. The minimal number of necessary acting elements in this universal computer is $N*2^N$ because each quantum gate has to act on each one of the N wavefunction-elements within each of the 2^N elements of the superposition. Sooner or later, it may be possible to build 100 functioning qubits; however, it will be impossible to build a *universal quantum computer* of 100 qubits computational capacity. Such a system would need $100*2^{100} \approx 10^{32}$ independent acting elements; this system cannot be built in the foreseeable future. If we suppose that the quantum gates are *single atoms*, and we use a densely packed monolithic silicon "chip" solid state quantum computer, then this 100 qubit computer chip would occupy a solid cube, over 12 meters on a side !!!

Some feature articles in the literature and magazines are saying that for a useful factorization algorithm, we would need a few thousand qubits. Inspired by these claims, let us then imagine the following universal quantum computer. A universal quantum computer chip would be a piece of monolithic solid, with the same volume as the Earth Globe. If we suppose again that the acting elements are the atoms, we end up with the surprising result: the capacity of this enormous system is only 158 qubits!

Thus, universal quantum computers with 100 qubits will remain science fiction; however, universal quantum computers with much smaller numbers of qubits are indeed possible. To get a feeling of what is allowed by microtechnology, let us continue these considerations.

Microprocessors using today's integration technology contain about 10^8 transistors and this number is presently doubling by a factor two every 18 months, in accordance with *Moore's Law*. Applying this trend to quantum computing technologies, using the most optimistic estimation (i.e. one element to be integrated for each acting element of a quantum gate) would result in only a very small quantum computer, ~ 22 qubits (we neglect any other elements needed for functionality). As long as *Moore's Law* holds, the number of qubits would increase one-by-one, every 18 months. Taking into the account the recent prediction of the end of *Moores Law*, we would never reach a 30 qubits universal computer.

Applying *Moore's Law* to wafer scale integration using 5mm thick 1 meter diameter silicon wafers and incorporating silicon atoms as stand-alone elements, the computer would have ~ 80 qubits and be implemented in the year 2100.

ii) Serial and, at the same time, statistical output port. *Are quantum computers really faster?*

Because of the Born interpretation and the collapse of the wavefunction, when we measure the superposition described by Eq. 1, we will get the eigenvalues of one of the superposition elements. However, we do not obtain the coefficient a_i values from only one measurement. The value $|a_i|^2$ will be obtained by reconstructing the superposition and repeating the measurement many times and thus empirically determining the probability $|a_i|^2$ of occurrence of its superposition element. In the optimal case (when the probabilities of the different coefficients are equal), for a relative error of Δ we need to repeat the whole process

$$\nu = \frac{1}{\Delta^2} 2^{2N} \qquad (4)$$

times. The practical problem comes in outputting the data. For example, for an accuracy of 0.1% of the value of $|a_i|^2$ we will need 10^6*2^{2N} repetitions of the process. At the 2 qubit case described by Eq. 2, we need to repeat the quantum calculation 16 million times to get the 4 numbers represented by these coefficients at an accuracy of 0.1%. If we use another quantum computer, and we want to extract a somewhat longer list of 1028 numbers as output data, the relevant qubit number at the output is N=10. Then the required number of repetitions of the calculation process for 0.1% accuracy is 1 billion!

iii) No cloning theorem: if a number has to be duplicated, it has to be measured by a serial, and at the same time, statistical inside port.

The title of the problem speaks for itself. We have the same problem as at the output, whenever we have to measure the data during the calculation process.

iv) Quantum decoherence. Extreme sensitivity to any kind of noise.

This is a topic which has been widely studied. The proposed solutions, error correcting codes, do not improve Shannon information but reduce speed. The only efficient solution so far seems to be using large quantum energies (photons) or extremely low temperatures.

v) The constraint of low temperatures.

Most quantum computer solutions need extremely low (sub-milliKelvin) temperatures to avoid decoherence (see iv above). Use of these temperatures would be very expensive and rules out the use of desktop, laptop and palmtop computers. At these temperatures, the thermal conductance of any solid material is extremely low. Therefore, even limited-speed data transfer from/to the quantum unit may cause enough power dissipation to warm it up to temperatures where the computer does not function properly. This fact may also impose further limits on input/output data speed.

3. ADVANTAGES OF HILBERT SPACE ANALOG (HSA) COMPUTERS

In this section, we address the corresponding properties of universal HSC chips based on analog electronic circuits (HSA computers) to be described in Section 4.

a) Complexity, size and speed.

HSA computers do have a real parallel output, not only quantum parallelism of the computation. All elements of the Hilbert space superposition can be read out simultaneously by parallel measurements. There is no collapse of the wavefunction using HSA processing. It is remarkable that with the same numbers of transistors as in today's PCs, such a HSA computer could simultaneously manipulate $\sim 10^7$ analog numbers corresponding to ~ 22 qubits, using quantum-parallel architecture. Analog computers have always been orders-of-magnitude faster than a corresponding digital computer with the same technology. The HSA computer combines this high-speed with other important characteristics, such as universality and quantum parallelism, which are missing from classical analog computers.

b) Parallel output port.

The data at the output are accessible in a parallel way and the access time is equal to the period time of the analog oscillators which can be as short as 0.1 nanosec (10^{-10} sec) using today's CMOS technology. The *real parallelism* of the output is the real stake here.

c) The no cloning theorem is irrelevant, data can be duplicated.

This title speaks for itself. Standard electronic circuitry is able to duplicate the data in a single follower amplifier step, with a very high accuracy.

iv) Quantum decoherence is irrelevant.

The noise of analog devices can be well controlled. No cooling or excessive energy dissipation is required.

v) HSA computers can work at room or elevated temperatures.

Because of this fact and the low power dissipation, desktop, laptop or palmtop design are possible.

4. ELEMENTS OF THE HSA COMPUTER

In this section, we very briefly describe a possible solution, which would make the development of a universal *HAQ* computer possible using standard analog computational elements. The circuits are designed to directly model the Hilbert space properties of quantum objects. The practical realization would probably be different; for example, integrator circuits would be used instead of time derivative circuits for a better stability and noise properties.

4.1 Generating the wavefunction, Fig.1. Using spins allows us to *fix the spatial coordinates*. The time-dependent part of the stationary Schrödinger equation $\breve{H}\psi(x_1,t) = i\hbar\frac{\partial\psi(x_1,t)}{\partial t} = E\psi(x_1,t)$ can be rearranged in the following way: $\frac{i\hbar}{E}\frac{\partial\psi(x_1,t)}{\partial t} = \psi(x_1,t)$. This equation is shown in an analog simulation in Fig. 1. The amplification is related to the energy eigenstate. Circuits providing inputs, initial conditions and coupling are not shown here.

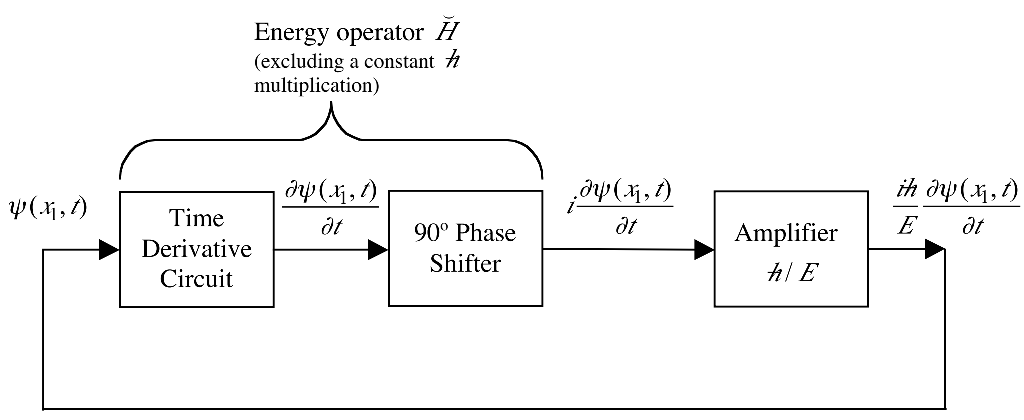

Figure 1. Simplified outline of the simulation of the time dependent part of the stationary solution.

4.2 The Spin Circuit; simulating a single spin, Fig. 2. A pair of such circuits can form a qubit. E_0 is the energy of the non-perturbed state. The spin variables (±1) of the local and neighboring spin introduce an energy change by the magnetic field, B, via the Bohr magneton μ_B and by spin-spin interaction via the coupling constant K.

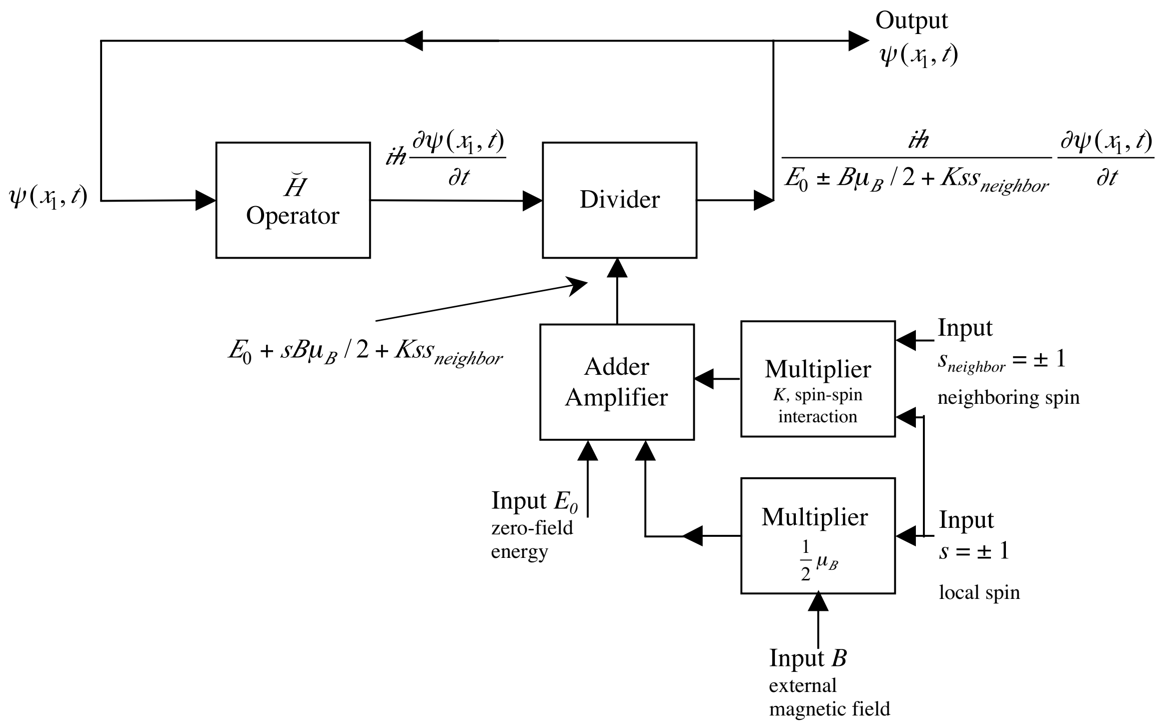

Figure 2. Spin Circuit, sketch of simulating a single spin.

4.3 The power spectrum of ψ at different cases, Fig. 3. The different spin orientations produce different energies corresponding to different oscillation frequencies. Here, spin interactions are neglected for simplicity. At $B \neq 0$, the spectral peaks are at different frequencies for different spin orientation, the corresponding wavefunctions are orthogonal, and the spin orientation is easy to measure/identify from the direct product (see next section). These wavefunctions form the basis of single spin Hilbert space.

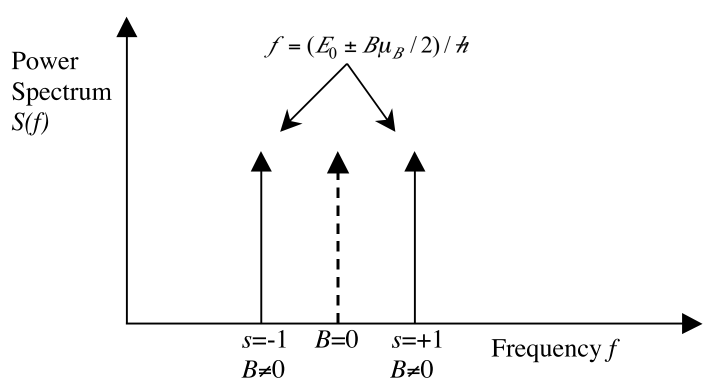

Figure 3. The power spectrum of ψ at different cases, demonstrating orthogonality.

4.4 Direct product and the quantum measurement, Fig. 4. Measurement of the spin direction is done using the direct product with the basis wavefunctions, spin-up and spin-down. The direct product of an N-dimensional Hilbert space vectors is calculated by the *parallel* application of N single units, one unit is used to each vector component pair.

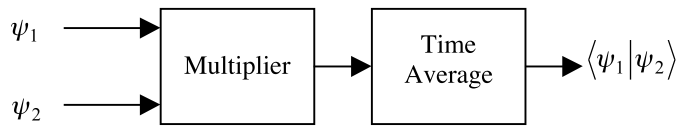

Figure 4. Direct product (one-dimensional) and the quantum measurement.

4.5. Single qubit circuit, Fig. 5. The circuit of Fig. 5 can also make a superposition of the states of a single spin. The two outputs represent the two elements of the Hilbert space related to a single qubit. If the qubit is used as a real qubit, then either C or D is zero and the other is 1 (no superposition).

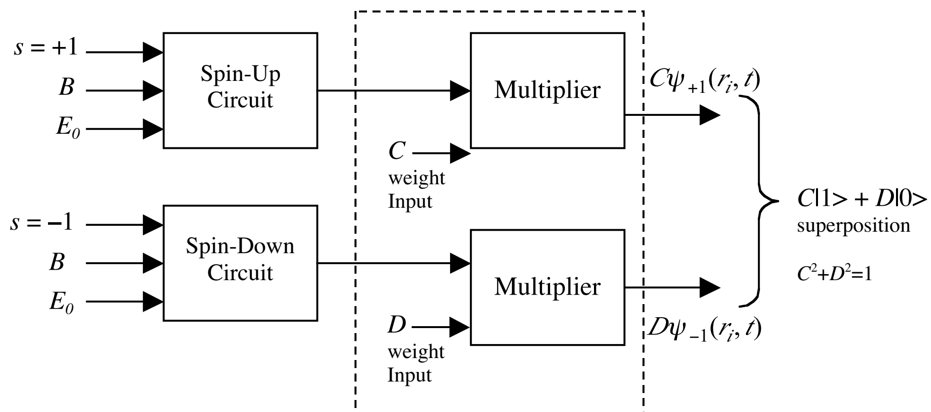

Figure 5. Single qubit circuit.

4.6 The general quantum system-state in the Hilbert space, Fig. 6. To represent the 2^N elements of the Hilbert space corresponding to N qubits, we need 2^N weighting units representing the space vectors and *each* weighting unit has to contain N outputs representing the substate of the N qubits in the corresponding product state. For example for two qubits, we have the following Hilbert space vectors:

$$|0,0\rangle , |1,1\rangle , |0,1\rangle , |1,0\rangle \quad (5)$$

Then the general state of the system will be:

$$Y = C|0,0\rangle + D|1,1\rangle + E|0,1\rangle + F|1,0\rangle \qquad C^2+D^2+E^2+F^2=1 \quad (6)$$

where the C,D,E,F weighting factors represent the 4 numbers the quantum computer will parallel-process. The advantage of the analog simulation method, compared to real quantum systems, is that the C,D,E,F numbers can be read out *parallel*, by a *single measurement*, with a high accuracy, without collapsing the wavefunction. Cloning of states without measuring them (that is making a direct product) can easily be done.

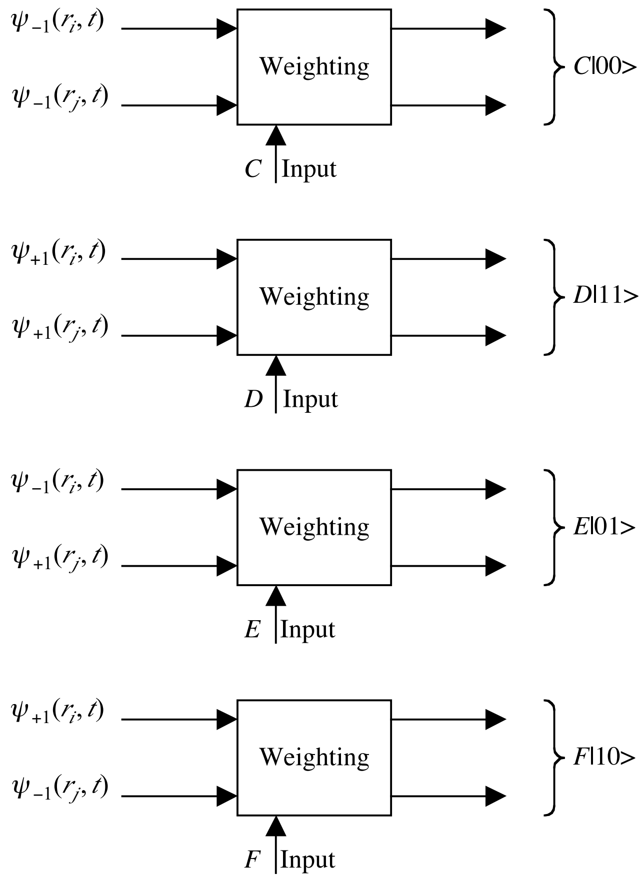

Figure 6. The general "quantum" system state in the 4-dimensional Hilbert space of two qubits.

4.7. The universal quantum gate Fig.7 and Fig. 8. To produce all possible unitary operations and entangled states and to build a universal quantum computer, it is enough to have universal quantum gates that consist of generalized rotations

$$V(\theta,\Phi)=\begin{pmatrix}\cos(\Theta/2) & -i\exp(-i\Phi)\sin(\Theta/2)\\ -i\exp(i\Phi)\sin(\Theta/2) & \cos(\Theta/2)\end{pmatrix} \tag{7}$$

and the XOR quantum gate[4]. Fig. 7 and 8 show the key elements needed to build the universal quantum gate.

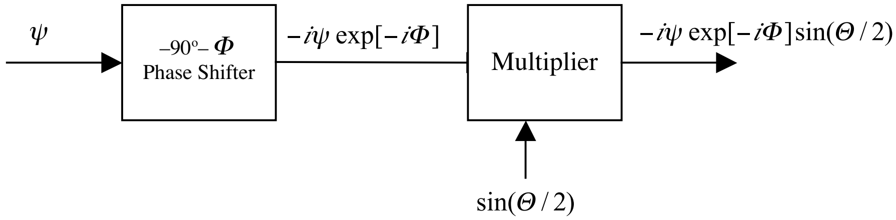

Figure 7. Key component of vector rotation, unitary operations and universal quantum gates.

An analog multiplier can easily do the cosine attenuation component of the rotation. The realization of the term containing the complex exponent and the sine component is shown on Fig. 7. The XOR quantum gate is shown in Fig. 8. The direct product circuits determine the spin state of the inputs. The circuitry uses also two classical logical gates. The gated amplifier transfers its input signal when a logical 1 is at its *Gate* input. Otherwise, its output is zero.

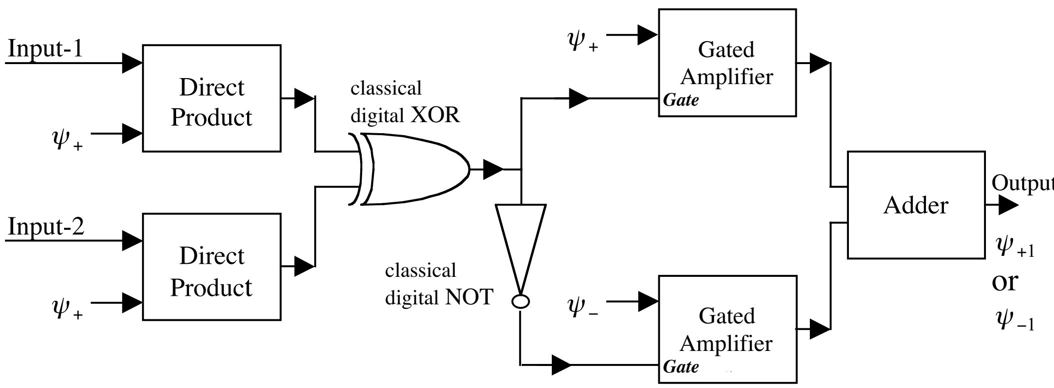

Figure 8. The other key component of universal quantum gates: the XOR operation.

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