Chapter 1
Introduction

This chapter introduces the fundamentals of diffractive optics, the similarities and differences between diffractive and refractive optics, the advantages of diffractive optics, and current challenges in this field. A quick review of the theoretical formulation of diffraction is presented, along with different theoretical approximations and their validity regimes.

1.1 Fundamentals of Diffractive Optics

1.1.1 Introduction

Diffraction was first observed by Francesco Maria Grimaldi in the year 1665. It was Grimaldi who first coined the term diffraction.[1] The study of diffraction was continued by Sir Isaac Newton,[2] James Gregory,[3] Thomas Young, etc.[4] Later, Augustin-Jean Fresnel used Huygens' wave principle to explain the diffraction phenomenon. Much later, scientists such as Poincaré, Sommerfeld, Kirchhoff, and Kottler, to name a few, added to the knowledge of the field.[5] Sommerfeld himself defined diffraction by what it was not, stating that diffraction could be considered to be any bending of rays not caused by refraction or reflection. Optical elements, surfaces, or interfaces change the behavior of light, or, in other words, change the basic properties of light, such as its amplitude, phase, direction, and polarization. These changes are brought about through the optical processes of refraction, reflection, interference, and diffraction. However, the amount of control and the ease of fabricating an element that exploits one or more of these processes vary. This book is about how to design diffractive elements that will modify some or all of these properties to create a desired behavior. The elements can be either reflective or transmissive in nature.

Diffraction, or, more correctly, diffractive optics, are now commonly used in many experimental and commercial systems.[6] The reasons for this are several: our use of light has gone far beyond illumination and communication. These, as well as the many other applications, such as imaging and sensing, require light to be manipulated in complicated ways, yet in compact systems. Diffractive optical elements (DOEs) are able to address both of these requirements simultaneously. With diffractive optics, as the name suggests, the main phenomenon used is diffraction rather than refraction. In this first chapter, we look at the transition between these effects. We examine the specific circumstances in which optics can be considered refractive or diffractive. This is important because the manner of describing them will be quite different. Most optical elements will exhibit a combination of refractive and diffractive properties. However, the dimensions of the element will determine the dominating phenomenon. By dimensions, we mean both the overall dimensions as well as the feature sizes. When feature sizes approach several wavelengths, an extended scalar theory is required, while for subwavelength features, a rigorous vector theory will be required to describe the element and its effect on light.[7]

In this book, we focus on elements that can be described by geometric optics and the scalar theory. No special software is required, and because feature sizes are relatively large, fabrication, too, is often simple. However, more importantly, scalar theory suffices in many situations and allows one to achieve fairly complex operations. The user will be introduced to different techniques that can be used to design and fabricate diffractive elements for specific applications. Special attention has been taken to present practical guidelines for fabrication. Since fabrication often requires the use of sophisticated, expensive equipment and consumables, it is prudent to carry out detailed simulations before actually fabricating. We also present well-commented programs in MATLAB® for direct use.

1.1.2 Refractive and diffractive optics

Diffraction is present in almost all phenomena involving light, although it may not always be dominant. Consider the case of focusing light with the aid of a refractive lens. Assuming the lens to be aberration-free, the smallest spot size obtained at the focal plane is called a diffraction-limited spot. This is because using conventional means it is impossible to focus light to a spot smaller than the diffraction-limited spot due to diffraction at the edges of the lens. With a refractive lens, image parameters are calculated based on geometric laws instead of diffractive principles as most of the incident light undergoes refraction, while only a small fraction of the input light undergoes diffraction. This is true with slits of dimensions much larger than the incident wavelength, as well. When the slit opening is large, it is a refraction-dominated system, and when it is smaller (with respect to the wavelength), it is a diffraction-dominated system, as shown in Fig. 1.1.

Diffraction can be qualitatively explained using Huygens' wave principle. Let us consider a plane wave. By Huygens' principle, every point on a wavefront acts as a source of secondary wavelets generating another plane

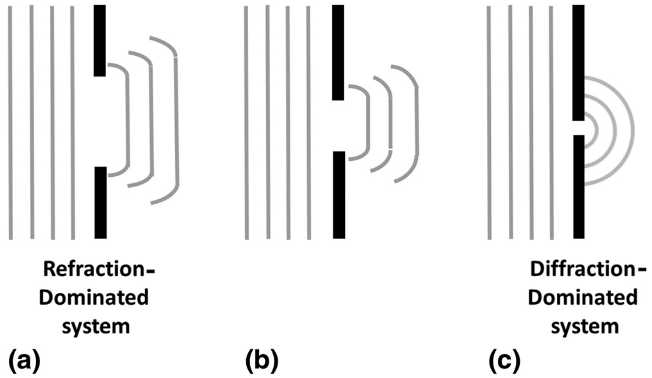

Figure 1.1 Diffraction of light in slits with different widths: (a)–(c) decreasing slit widths show increasing domination of diffraction.

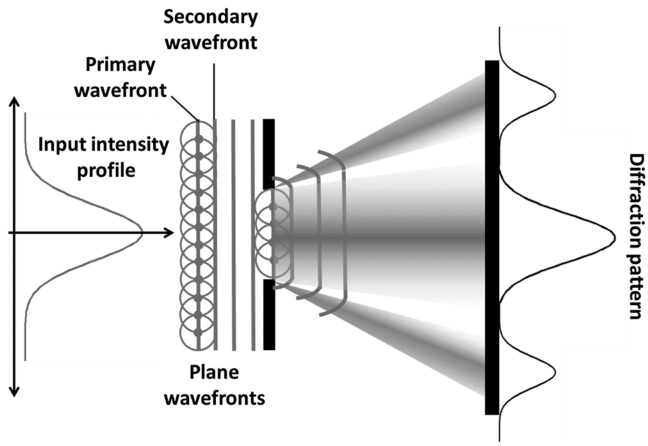

Figure 1.2 Diffraction of a plane wavefront with a Gaussian intensity profile at a single-slit aperture.

wavefront. However, when part of the wavefront is blocked by a slit, the wavefront bends at the edges, as shown in Fig. 1.2.

In refraction, light can be thought of as traveling in straight lines in a medium of constant refractive index. Snell's law applies at interfaces (surfaces where the refractive index changes) and can be used to determine the new direction. Refractive elements, in general, consist of a single bulk unit, whose shape and refractive index determine its imaging properties. Diffractive elements, unlike their refractive counterparts, are made of many different zones. The final image is a coherent superposition of light diffracted from the various zones. Every point on the aperture contributes to the intensity at one location of the output. Of course, refraction will also take place. The resulting behavior will, therefore, be a combination of both effects. For example, the 0^{th} diffraction order of a reflection grating is nothing but the light that obeys geometric optics laws. It should be clear that although diffraction is an interference effect, these two effects are distinguished from each other by the number of interacting beams. Most interferometers, for example, will create two beams that interfere later in the optical path. In diffraction, an infinite number of beams play a role.

The well-known Fresnel lens was invented by Augustin-Jean Fresnel in 1822 to replace the heavy, conventional spherical lenses used in lighthouses.[7] Fresnel converted the bulk conventional spherical lens shown in Fig. 1.3(a) into a thin Fresnel lens by arranging different sections of the conventional lens in a plane, as shown in Fig. 1.3(b). The curvature of the material at the glass–air interface and the refractive index of the glass material govern the light-bending profile; hence, the inactive glass material present in the conventional lens can be removed without altering the function of the device. In this example, the Fresnel lens has been made by sectioning the original lens into three horizontal parts. It behaves almost like the conventional lens except for extra diffraction effects occurring at the boundaries between its different sections. What is important is that the Fresnel lens is still a refractive optical element with dimensions of t and d much greater than λ, the wavelength of the incident light. However, if the same element were to be designed with feature sizes closer to the wavelength of light, then the resulting element would be predominantly diffractive.[8] In the latter case, even though the function of the lens remains the same, wavefront control is achieved by diffraction rather than refraction.

From the above discussion, it seems that it is relatively easy to design a diffractive element from the shape information of an equivalent refractive optical element (ROE), assuming that such an element exists. Let us study this concept in some more detail.

In refractive optics, the bending of light occurs due to the geometry of the structure and the index of refraction, while in diffractive optics, bending of light occurs due to the features and the aperture edges. Given a particular intensity distribution across an aperture, it is possible to design the features

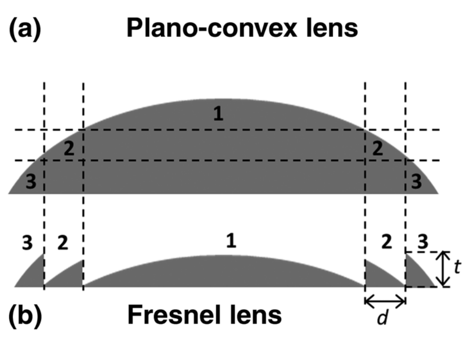

Figure 1.3 Scheme showing the generation of a Fresnel lens from a conventional plano-convex lens: (a) conventional lens and (b) Fresnel lens. If t and $d \gg \lambda$, the lens is refractive. If t and d are on the order of, or less than, the wavelength, the element is diffractive.

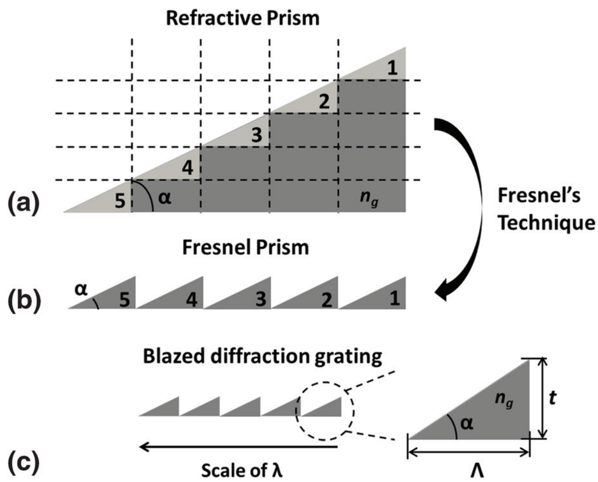

Figure 1.4 (a) Refractive prism and (b) generation of a Fresnel prism from it. (c) Blazed diffraction grating with dimensions on the order of the wavelength.

that will fill the aperture to obtain a desired intensity distribution at an output plane. The first DOEs were modified versions of refractive elements with feature sizes on the order of the incident wavelength. To understand the above statement, let us consider the conversion of a prism into a diffraction grating. In the refractive regime, a prism can be used to disperse light or change the direction of an incident monochromatic light. A similar element in the diffractive regime is a grating.

The construction of a diffraction grating from a prism is shown in Fig. 1.4. The base angle of the prism and its refractive index are given by α and n_g, respectively. The refractive bulk prism is converted into a thin element using Fresnel's technique, as shown in Figs. 1.4(a) and (b). Figure 1.4(c) shows a diffraction grating, which is similar to Fig. 1.4(b), except that the feature sizes are closer to the wavelength of light with a period Λ and thickness t. A Fresnel prism is a refraction-dominated system, while a diffraction grating is a diffraction-dominated one. A major difference arising because of this is the fact that the former will bend the incident beam into one direction, while the latter will generate multiple orders. The shape of the diffraction grating will determine the number of orders and will be discussed in more detail in later chapters. In order to force most of the diffracted light into a single diffraction order, the diffraction grating must be blazed (i.e., have a triangular shape) with a height or thickness t given by

$$t = \frac{\lambda}{(n_g - 1)}, \tag{1.1}$$

which corresponds to a phase difference of 2π.[9] Hence, the relationship between α of the prism and Λ of the grating for normal incidence of light is given by

$$\alpha = \tan^{-1}\left[\frac{\lambda}{\Lambda(n_g - 1)}\right]. \tag{1.2}$$

Equation (1.2) shows that the geometrical profile of the prism is related to the period of the diffraction grating.

The deviation angle β of the prism with a base angle of α and refractive index n_g can be calculated using trigonometry as

$$\beta = \sin^{-1}(n_g \sin\alpha) - \alpha. \tag{1.3}$$

In order to become familiar with the terms and the language of diffractive optics, we briefly introduce the amplitude grating here. Many of the concepts will directly hold true for a phase grating as well. The second chapter provides a much more detailed look at gratings and methods by which to design and simulate their behavior.

Imagine a structure comprising a number of reflective slits surrounded by opaque regions, as shown in Fig. 1.5. The slits are periodically spaced with a distance Λ. This structure defines a basic diffraction grating. Light is incident at an angle of β_i with respect to the grating normal, which is indicated as vertical dashed lines in the figure. The question is what determines the angle(s) β_r of the beam after incidence on this surface? Since we have chosen reflective slits, the light will travel back into the region of incidence, but we will not call this reflection as the resulting intensity is due to the superposition of many beams. For a more detailed description, we refer readers to a number of books that discuss Huygens' principle (every point in the slit acts a secondary source) and scalar diffraction.[1–5] Suffice to say that the multitudes of beams from each slit interact with each other and result in an intensity pattern in the far field. This pattern is not uniform, and the goal is to determine the locations of the intensity peaks.

To arrive at the pattern, we look at two rays AB and $A'B'$ that both originated from the wavefront AA'. In other words, at the plane AA', both rays started with the same phase. For the diffracted wave shown in the figure, to represent an actual wave, BB' should be a wavefront. That is, the path

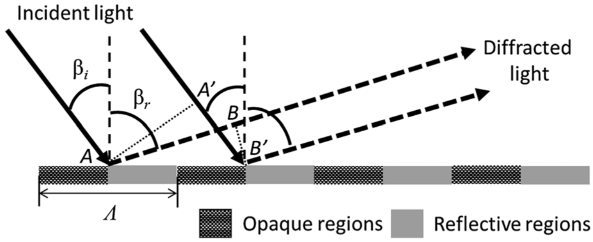

Figure 1.5 Schematic of light diffraction in a reflective amplitude grating.

length difference between AB and $A'B'$ must be equal to a multiple of the wavelength λ, as described in

$$n_1 A'B' - n_2 AB = m\lambda_0, \tag{1.4}$$

where, n_1 and n_2 are the refractive indices seen by the incident and reflected rays, respectively. Since both are in the same medium, $n_1 = n_2$. With this information, and comparing the triangles $AA'B'$ and $BB'A$, the equation can be rewritten as

$$\Lambda(\sin\beta_i - \sin\beta_r) = m\lambda_0, \tag{1.5}$$

where, $\lambda = \lambda_0/n_1$, and m is the order number. The implication of the order parameter m is that Eq. (1.5) is satisfied for different values of β_r. Therefore, the equation could be more accurately be written as

$$\Lambda(\sin\beta_i - \sin\beta_{mr}) = m\lambda, \tag{1.6}$$

where, β_{mr} represents the m^{th} diffraction order. Thus, a picture of what is happening after diffraction from the grating slowly emerges. Unlike specular reflection, where the reflected light travels in one direction only, or scattered light disperses into a solid angle from a surface, several 'diffraction' orders exist. One could think of this as reflection occurring in a finite number of preferred directions. If the grating had been a transmissive one, then refraction would occur in more than one direction. The condition $m = 0$ represents the classical optics case. For example, in the above grating, the condition $m = 0$ results in $\beta_i = \beta_r$, which is the law of reflection. For a transmission grating, $m = 0$ would reduce the equation to Snell's law. While the equation allows us to predict the possible directions of travel, it gives us no information about how much light travels in each order. This means that we cannot predict the efficiency of the diffractive structure. Obviously, efficiency is important, and later chapters will include further equations that can be used during the design stage to maximize it. The convention used to name the various orders of a grating is indicated in Fig. 1.6. The figure can be used for either a transmission or reflection grating. Angles are always measured from the grating normal. The sign of the angle depends on the direction of rotation of the ray from the normal (indicated by $\pm$ signs in the figure). On the other hand, the sign of the order parameter m depends on the direction from the 0^{th} order. For example, for the reflected $m = +1$ order in the figure, angle β_1 is positive but would have been negative if the ray happened to lie on the other side of the normal.

For simplicity, normal incidence is considered, and for the 1^{st} diffraction order, Eq. (1.6) can be simplified. The diffraction angle β_1 of the 1^{st} diffraction order of the diffraction grating with a period of Λ is given by

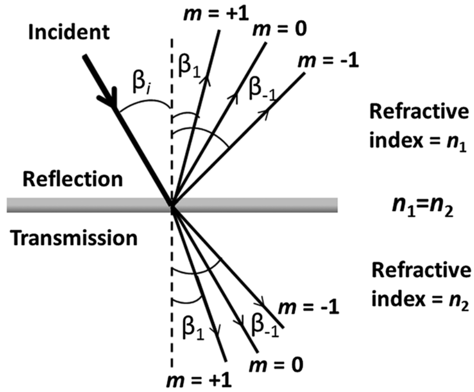

Figure 1.6 Orders of a grating.

$$\beta_1 = \sin^{-1}\left(\frac{\lambda}{\Lambda}\right). \tag{1.7}$$

By substituting Eqs. (1.1) and (1.2) in (1.7), β_1 can be expressed as

$$\beta_1 = \sin^{-1}[(n_g - 1)\tan\alpha]. \tag{1.8}$$

From Eqs. (1.3) and (1.8), for cases where the base angle of prism α and the refractive index n_g are small, or where the period of the diffraction grating Λ is large, Eqs. (1.3) and (1.8) reduce to a simpler equation:

$$\beta_1 = \beta = (n_g - 1)\alpha. \tag{1.9}$$

This is an interesting result. When the period of the diffraction grating is large, it behaves more like a refractive element; however, its behavior is different when Λ approaches λ. To quantitatively understand this, a few typical cases are considered with $n_g = 1.1$, 1.5, and 1.9. The deviation angles β of a prism and diffraction angles β_1 of a grating were calculated using Eqs. (1.3) and (1.4), respectively, and plotted against base angles α of a prism, as shown in Fig. 1.7.

For smaller values of α, there is good overlap between β and β_1. Therefore, for DOEs with small diffraction angles, it is possible to derive the profile blueprint from a ROE with an equivalent function. Examples of some other elements that can be achieved in a similar way are the axicon,[9] circular grating,[10] ring lens,[11] and diffractive ring lens.[12] We must however, always keep in mind that in any optical element, both refraction and diffraction coexist. The dominating effect, dictated by the feature sizes, decides whether the element is diffractive or refractive.

Several researchers have studied the transition between refractive and diffractive elements quite extensively.[13,14] As discussed, this can be done by

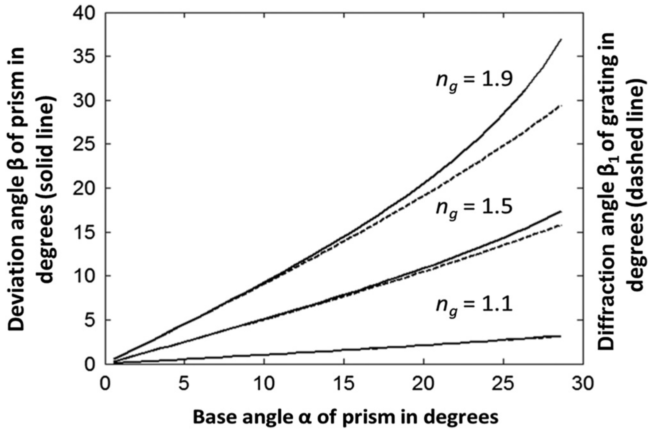

Figure 1.7 Plot of deviation angle β of a prism (solid line) and diffraction angle β_1 of a grating (dashed line) versus variations in the base angle α of the prism for $n_g = 1.1$, 1.5, and 1.9.

changing the feature sizes of an element and studying its behavior as the size changes. In particular, the dispersive nature will vary depending on the feature size. Again, let us take the examples of a lens and a prism. In the latter case, we will compare a 1D (diffractive) grating and a (refractive) prism, as both result in an off-axis deflection of the incident beam.[14] In both cases, the refractive index and, hence, the wavelength plays a role in the amount of deflection. It is shown in Ref. 14 that the nature of dispersion is quite different for the refractive and diffractive cases, with the latter experiencing much greater dispersion for the same angle of deflection. Even more interesting is the negative sign of the grating dispersion compared to that of the prism. In other words, when light bends, different wavelengths bend by different amounts, and the direction of bending is determined by the base optical behavior of the element. The opposite signs of the dispersion of refraction and diffraction have been used from very early on to provide some amount of achromatization.[15] Recent publications show that this concept is still being manipulated for achromatization.[16,17] In the refractive case, dispersion caused by the material dominates, whereas, the structure of the element controls the diffractive dispersion. Given these two very different causes, diffraction and refraction cannot be balanced in a single element. Researchers are now studying harmonic DOEs that lie somewhere between these two distinct effects.[18] In conclusion, it is clear that the structure of a DOE can be deduced from the structure of a ROE that performs a similar optical operation.

1.1.3 Scalar diffraction formulation

Diffraction is a phenomenon that is observable due to the wave nature of light. The effects of diffraction are more noticeable when light interacts with an interface or an optical element whose dimensions are close to that of the incident wavelength. Diffraction theory allows one to calculate how wavefronts change and how they travel after interaction with such an element. A complete analysis of a diffractive system can be carried out using the vector diffraction equations.[19,20] Vector diffraction formulation can be used to understand the behavior of DOEs with features both superwavelength as well as subwavelength. This formulation can determine not only the intensity and phase profiles at different planes, but also the state of polarization at these planes. However, the formulation is quite difficult to implement and also to simulate. For most of the analysis, which does not involve DOEs with features smaller than or equal to the wavelength of light, and does not need to explain the polarization state of the diffracted field, the simpler scalar diffraction formulation is sufficient. The focus of this text book is only on superwavelength DOEs; therefore, the discussion is limited only to the scalar diffraction formulation. Few models have been developed that can explain the polarization state of a diffracted field using only scalar diffraction formulation.[21] The scalar diffraction formulation is described in numerous text books.[4,5] With the assumption that the features of the DOE are larger than the wavelength of the source, and for spherical wavefronts, the scalar diffraction formula based on Huygens–Fresnel theory is given by

$$E(u,v) = \frac{z}{\lambda j} \iint_{\Sigma} A(x,y) \frac{\exp(jkr)}{r^2} \, dx dy, \tag{1.10}$$

where (x, y) is the diffraction plane, and (u, v) is the observation plane. The radius r can be given by

$$r = \sqrt{z^2 + (u-x)^2 + (v-y)^2}. \tag{1.11}$$

For smaller angles, spherical wavefronts can be approximated as parabolic wavefronts, which results in the Fresnel approximation formula. Now the radius can be approximated as

$$r \approx z\left[1 + \frac{1}{2}\left(\frac{u-x}{z}\right)^2 + \frac{1}{2}\left(\frac{v-y}{z}\right)^2\right]. \tag{1.12}$$

The Fresnel diffraction formula is given by

$$\begin{aligned} E(u,v) = \frac{e^{jkz} e^{j\frac{k}{2z}(x^2+y^2)}}{j\lambda z} \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} \left\{ A(x,y) \exp\left[j \frac{k}{2z} (x^2+y^2) \right] \right\} \\ \times \exp\left[-j \frac{2\pi}{\lambda z} (xu + yv) \right] dx dy. \end{aligned} \tag{1.13}$$

It can be seen that the Fresnel diffraction formula is relatively simpler to solve (as is simulating the diffraction field) compared to the prior

Huygens–Fresnel approximation. However, this approximation will fail in the region closer to the diffraction plane and for large diffraction angles. Equation (1.13) is valid only when

$$z^3 \gg \frac{\pi}{4\lambda}\left[(x-u)^2+(y-v)^2\right]^2_{\max}. \tag{1.14}$$

Equation (1.13) for large values of z reduces to just a Fourier transform operation, which is called the far-field approximation. This approximation, also known as the Fraunhofer approximation, is valid only for very large distances given by

$$z \gg \frac{k(u^2+v^2)}{2}. \tag{1.15}$$

At these distances, the radius of the spherical wavefront is large, so a section of the spherical wavefront can be approximated to be a plane wavefront. A summary of the approximations of the scalar diffraction formulation is shown in Fig. 1.8. In this book, only the Fresnel and Fraunhofer approximations are used for analysis of DOEs. The scalar diffraction integrals can be thought of as

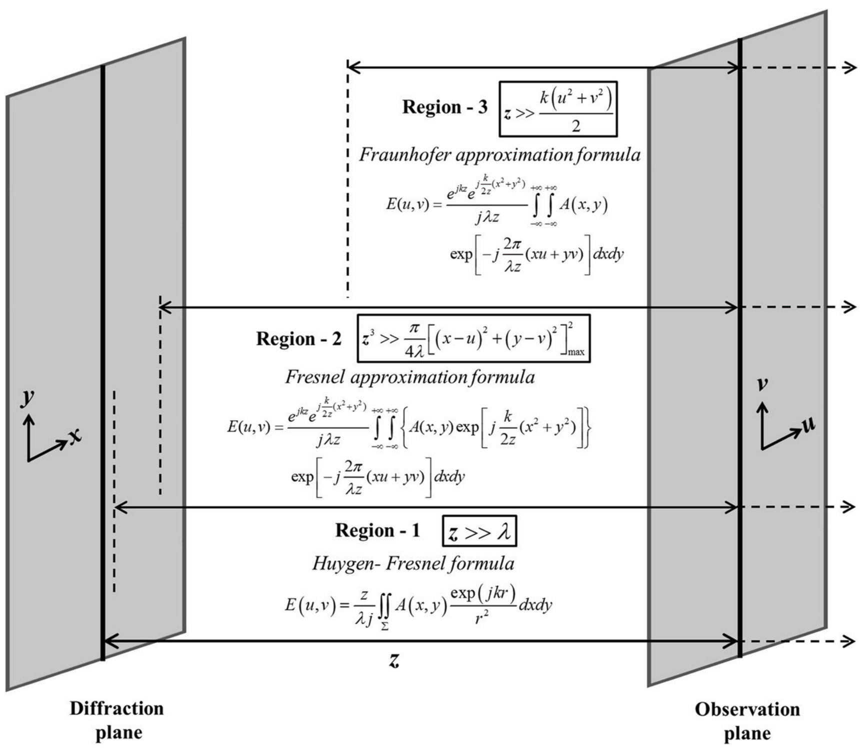

Figure 1.8 Depiction of the validity of different approximations of the scalar diffraction formula.

scalar diffraction propagators. In other words, they are a means to find the diffraction amplitude and phase at any plane, given these values at an earlier plane. These integral equations are used in their analog continuous form only when analytic solutions for the diffraction problem being studied exist. For all other cases, different techniques can be used to solve them to determine the diffraction field at some plane. While the similarity to a Fourier transform (FT) is clear when studying Fraunhofer diffraction, even diffraction at closer planes can use the FT concept, as is obvious from Eq. (1.13). (Further details are provided in Chapter 4.) Therefore, one common method to solve scalar diffraction integral equations uses discretized Fourier transforms (DFTs). One particularly efficient and fast algorithm that implements a DFT is known as the fast Fourier transform (FFT). This algorithm carries out the FT operation for N discrete samples in $O(N \log N)$ steps, rather than the $O(N^2)$ steps of a standard FT operation. Algorithms that carry out a DFT with $O(N^2)$ steps can also be used. These algorithms have other benefits; for example: the size of the matrices (arrays) used does not need to be powers of 2; there is more freedom in choosing matrix sizes (hence, more freedom in fixing resolution at the diffraction plane); they can tackle problems that cannot be handled by the FFT algorithm, etc. While these benefits may seem attractive, they come at the price of longer computation times. In addition to techniques that use the FT as a basis for a diffraction solution, other beam propagation techniques such as wavelets,[22] finite element methods,[23] etc.,[24–26] can be used.

1.2 Software for Designing Diffractive Optics

In the preface, the importance of diffractive optics was discussed. Given its many uses, clearly, the ability to design and simulate DOEs is critical. Most researchers use their own programming scripts to do this. And, of course, the goal of this book is to help such a researcher. For the sake of completeness, however, we mention other resources[57–59] that are available.

The company Wyrowski Photonics UG[27] markets a software package called VirtualLab Fusion.[28] Its diffractive optics toolbox can be used for the generation of micro- and diffractive optical elements. With this software, a variety of DOEs such as beam shapers, splitters, and diffusers can be designed. It also has a grating toolbox with which rigorous analysis of grating can be carried out. VirtualLab Fusion can then be used to analyze imaging in systems containing gratings and diffractive or hybrid lenses.

GSolver,[29] on the other hand, is a software that allows rigorous analysis of all types of diffractive gratings. It provides a vector solution for complex periodic grating structures.

DiffractMOD™[30] is used to model a wide range of devices including diffractive optics, such as diffractive optical elements, subwavelength periodic structures, and photonic bandgap crystals. It is based on rigorous coupled-wave analysis (RCWA) and can handle both metallic and dielectric materials, allowing for the inclusion of plasmonic effects as well.

Even software like OSLO[31] and Zemax[32] that typically are used for modeling and designing refractive optics allow some amount of diffractive modeling. They do so without using any of the diffraction equations and, therefore, can most easily deal with relatively simple periodic diffractive elements. Diffraction is included, taking into account the fact that a diffractive surface introduces additional ray bending over what a refractive element would achieve.

While a variety of software tools exist, the authors believe that much of the required modeling can be done by the users themselves, with software such as MATLAB, Scilab, Python, or C. This is especially true for scalar diffractive optics. The advantage, apart from cost, is that design and simulation programs can be tuned exactly to the users' requirements.

1.3 Concluding Remarks

A brief history of diffraction and the similarities and differences between DOEs and ROEs are presented in the previous sections with a glimpse of the fundamental scalar diffraction formula. In this concluding section, the scope of research and development in diffractive optics is summarized followed by the contents of the following chapters.

DOEs in general are smaller and thinner compared to their refractive equivalents.[33] Additionally, DOEs can be engineered to nanometer accuracy due to the remarkable growth in the field of micro/nanolithography and fabrication techniques.[34] DOEs can be fabricated with feature sizes from few hundreds of nanometers to few millimeters. Extremely fine features smaller than the diffraction limit of light can be achieved using extreme ultraviolet lithography,[35] electron beam lithography,[36,37] and focused ion beam lithography.[38,39] The transfer of smaller features to glass and hard substrates can be carried out using sophisticated etching processes;[40,41] therefore, elements that can withstand higher optical powers can be fabricated. DOEs can also be designed and implemented for other parts of the electromagnetic spectrum, such as x rays, etc.[42,43] Hence, DOEs can replace refractive optics in various applications.[44]

In many optical setups with refractive optical elements,[45] some elements are paired without any relative motion between them. In such cases, it is often convenient to replace these elements with one DOE with equivalent functionality (of the replaced elements).[46] Multiple functions such as beam re-orientation, focusing, and splitting have been reported.[47,48] Hence, it is possible to convert bulky optical systems into lightweight, compact systems with high-quality beam profiles in less space. In the case of DOEs, the resolution of the element can be less than 10 nm, which is at least three orders of magnitude higher than that of conventional spatial light modulators; in addition, DOEs are lighter. Clearly, one can obtain a resolution better than that of refractive elements while maintaining a compact optics configuration. Besides the above advantages, some beam profiles, such as vortex beams, chiral beams, etc.,[49–52] cannot be generated using ROEs. Therefore, it is possible to revolutionize the field of optical instrumentation by replacing ROEs with equivalent lightweight DOEs and DOE- based optical instruments. Recent research reports the fabrication of DOEs on the tip of optical fiber, where the patterned fiber can be attached directly to a fiber laser to achieve high-power beam shaping.[53] These results could be useful for a wide variety of biomedical applications such as laser-based surgery,[54] endoscopy,[55] laparoscopy,[56] etc.

References

1. A. Authier, *Early Days of X-Ray Crystallography*," Oxford University Press, Oxford (2013).
2. J. Z. Buchwald and I. B. Cohen, Eds., *Isaac Newton's Natural Philosophy*, MIT Press, Cambridge, Massachusetts (2001).
3. S. Singh, *Fundamentals of Optical Engineering*, Discovery Publishing House Pvt. Ltd, New Delhi (2009).
4. E. Hecht, *Optics*, Fourth Edition, Addison Wesley, San Francisco (2002).
5. J. W. Goodman, *Introduction to Fourier Optics*, Second Edition, McGraw-Hill Companies Inc., New York (1996).
6. H. P. Herzig, *Micro-optics: Elements, Systems and Applications*, Taylor and Francis Ltd., London (1997).
7. Y. G. Soskind, *Field Guide to Diffractive Optics*," SPIE Press, Bellingham, Washington (2011) [doi: 10.1117/3.895041].
8. K. Miyamoto, "The phase Fresnel lens," *J. Opt. Soc. Am.* **51**, 17–20 (1961).
9. J. H. McLeod, "The axicon: a new type of optical element," *J. Opt. Soc. Am.* **44**, 592–592 (1954).
10. J. Dyson, "Circular and spiral diffraction gratings," *Proc. R. Soc. London Ser. A* **248**, 93–106 (1958).
11. J. B. Goodell, "Eccentric lenses for producing ring images," *Appl. Opt.* **8**, 2566–2566 (1969).
12. M. R. Descour, D. I. Simon, and W.-H. Yei, "Ring-toric lens for focus-error sensing in optical data storage," *Appl. Opt.* **38**, 1388–1392 (1999).
13. I. Snigireva, A. Snigirev, C. Rau, T. Weitkamp, V. Aristov, M. Grigoriev, S. Kuznetsov, L. Shabelnikov, V. Yunkin, M. Hoffmann, and E. Voges, "Holographic x-ray optical elements: Transition between refraction and diffraction," *Nucl. Instrum. Methods Phys. Res. Sect. A* **467-468**, 982–985 (2001).

14. S. Sinzinger and M. Testorf, "Transition between diffractive and refractive micro-optical components," *Appl. Opt.* **34**, 5970–5976 (1995).
15. T. Stone and N. George, "Hybrid diffractive-refractive lenses and achromats," *Appl. Opt.* **27**, 2960–2971 (1988).
16. Y. Yoon, "Design and tolerancing of achromatic and anastigmatic diffractive–refractive lens systems compared with equivalent conventional lens systems," *Appl. Opt.* **39**, 2551–2558 (2000).
17. P. Valley, N. Savidis, J. Schwiegerling, M. R. Dodge, G. Peyman, and N. Peyghambarian, "Adjustable hybrid diffractive/refractive achromatic lens," *Opt. Express* **19**, 7468–7479 (2011).
18. D. W. Sweeney and G. E. Sommargren, "Harmonic diffractive lenses," *Appl. Opt.* **34**, 2469–2475 (1995).
19. A. S. Marathay and J. F. McCalmont, "Vector diffraction theory for electromagnetic waves," *J. Opt. Soc. Am. A* **18**, 2585–2593 (2001).
20. A. Wang and A. Prata, "Lenslet analysis by rigorous vector diffraction theory," *J. Opt. Soc. Am. A* **12**, 1161–1169 (1995).
21. M. Mansuripur, "Distribution of light at and near the focus of high-numerical-aperture objectives," *J. Opt. Soc. Am. A* **3**, 2086–2093 (1986).
22. F. T. S. Yu, S. Jutamulia, and S. Yin, Eds., *Introduction to Information Optics*, Academic Press, San Diego (2001).
23. D. W. Prather and S. Shi, "Formulation and application of the finite-difference time-domain method for the analysis of axially symmetric diffractive optical elements," *J. Opt. Soc. Am. A* **16**, 1131–1142 (1999).
24. G. N. Nguyen, "Modeling, Design and Fabrication of Diffractive Optical Elements Based on Nanostructures Operating Beyond the Scalar Paraxial Domain," Doctoral Dissertation, Télécom Bretagne; Université de Bretagne Occidentale, France (2014).
25. Q. Cao and J. Jahns, "Comprehensive focusing analysis of various Fresnel zone plates," *J. Opt. Soc. Am. A* **21**, 561–571 (2004).
26. V. A. Soifer, *Diffractive Nanophotonics*, CRC Press, Boca Raton, Florida (2014).
27. J. Turunen and F. Wyrowski, *Diffractive Optics for Industrial and Commercial Applications*, John Wiley & Sons, Hoboken, New Jersey (1997).
28. http://www.wyrowski-photonics.com/virtuallab/toolboxes.html
29. http://www.gsolver.com/
30. https://optics.synopsys.com/rsoft/rsoft-passive-device-diffractMOD.html
31. http://www.lambdares.com/oslo
32. https://www.zemax.com/os/resources/learn/knowledgebase/how-to-design-diffractive-optics-using-the-binary
33. B. C. Kress and P. Meyrueis, *Applied Digital Optics*, John Wiley & Sons, Chichester, UK (2009).

34. M. T. Gale, "Replication techniques for diffractive optical elements," *Microelectron. Eng.* **34**, 321–339 (1997).
35. B. Wu and A. Kumar, "Extreme ultraviolet lithography: A review, *J. Vac. Sci. Technol. B* **25**, 1743–1761 (2007).
36. A. N. Broers, A. C. F. Hoole, and I. M. Ryan, "Electron beam lithography: Resolution limits," *Microelectron. Eng.* **32**, 131–142 (1996).
37. V. R. Manfrinato, L. Zhang, D. Su, H. Duan, R. G. Hobbs, E. A. Stach, and K. K. Berggren, "Resolution limits of electron-beam lithography toward the atomic scale, *Nano Lett.* **13**, 1555–1558 (2013).
38. J. Melngailis, "Focused ion beam technology and applications, *J. Vac. Sci. Technol. B* **5**, 469–495 (1987).
39. F. Watt, A. A. Bettiol, J. A. V. Kan, E. J. Teo, and M. B. H. Breese, "Ion beam lithography and nanofabrication: A Review," *Int. J. Nanosci.* **4**, 269–286 (2005).
40. S. Ronggui and G. C. Righini, "Characterization of reactive ion etching of glass and its applications in integrated optics," *J. Vac. Sci. Technol. A* **9**, 2709–2712 (1991).
41. L. Li, T. Abe, and M. Esashi, "Smooth surface glass etching by deep reactive ion etching with SF6 and Xe gases," *J. Vac. Sci. Technol. B* **21**, 2545–2549 (2003).
42. W. Chao, J. Kim, S. Rekawa, P. Fischer, and E. H. Anderson, "Demonstration of 12 nm resolution Fresnel zone plate lens based soft x-ray microscopy," *Opt. Express* **17**, 17669–17677 (2009).
43. A. Takeuchi, K. Uesugi, H. Takano, and Y. Suzuki, "Submicrometer-resolution three-dimensional imaging with hard x-ray imaging micro-tomography," *Rev. Sci. Instrum.* **73**, 4246–4249 (2002).
44. S. Sinzinger and M. Testorf, "Transition between diffractive and refractive micro-optical components," *Appl. Opt.* **34**, 5970–5976 (1995).
45. P. A. Belanger and M. Rioux, "Ring pattern of a ring-axicon doublet illuminated by a Gaussian beam," *Appl. Opt.* **37**, 7049–7054 (1978).
46. A. Vijayakumar, "Design, Fabrication, and Evaluation of Diffractive Optical Elements for the Generation of Focused Ring Patterns," Doctoral Thesis, Indian Institute of Technology Madras, Chennai, India (2015) [doi: 10.13140/RG.2.2.25996.51847].
47. Y. Backlund, J. Bengtsson, and A. Larsson, "Waveguide hologram for outcoupling and simultaneous focusing into multiple arbitrary positions," *IEEE Photonics Technol. Lett.* **10**, 1286–1288 (1998).
48. Y. Backlund, J. Bengtsson, C.-F. Carlstrom, and A. Larsson, "Multi-functional grating couplers for bidirectional incoupling into planar waveguides," *IEEE Photonics Technol.* **12**, 314–316 (2000).
49. G. Indebetouw, "Optical vortices and their propagation," *J. Mod. Opt.* **40**, 73–87 (1993).

50. I. V. Basistiy, M. S. Soskin, and M. V. Vasnetsov, "Optical wavefront dislocations and their properties," *Opt. Commun.* **119**, 604–612 (1995).
51. M. S. Soskin and M. V. Vasnetsov, "Singular optics," *Prog. Opt.* **42**, 219–276 (2001).
52. M. R. Dennis, R. P. King, B. Jack, K. O. Holleran, and M. J. Padgett, "Isolated optical vortex knots," *Nat. Phys.* **6**, 118–121 (2010).
53. V. Pramitha, S. Bhattacharya, U. Eigenthaler, K. Keskinbora, C. T. Samlan, M. Hirscher, J. P. Spatz, and N. K. Viswanathan, "Direct patterning of vortex generators on a fiber tip using a focused ion beam," *Opt. Lett*. **41**, 2133–2136 (2016).
54. J. R. Leger, D. Chen, and Z. Wang, "Diffractive optical element for mode shaping of a Nd:YAG laser," *Opt. Lett.* **19**, 108–110 (1994).
55. B. A. Flusberg, E. D. Cocker, W. Piyawattanametha, J. C. Jung, E. L. M. Cheung, and M. J. Schnitzer, "Fiber-optic fluorescence imaging," *Nature Methods* **2**, 941–950 (2005).
56. Y. Qin, H. Hua, and M. Nguyen, "Multiresolution foveated laparoscope with high resolvability," *Opt. Lett.* **38**, 2191–2193 (2013).
57. S. Sinzinger and J. Jahns, *Microoptics*, Second Edition, Wiley-VCH, Weinheim (2003).
58. J. Jahns and S. Helfert, *Introduction to Micro- and Nanooptics*, Wiley-VCH, Weinheim (2012).
59. H. Zappe, *Fundamentals of Micro-Optics*, Cambridge University Press, Cambridge (2010).